Targets & Resources: A Screening Perspective*

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Abstract

To secure funding for a project, an agent (informed about the project's type) announces a target output. The principal provides more generous resources for high targets but makes compensation tied to performance relative to the projection. The incentive mechanism is geared towards screening project/agent types for resource disbursement at the ex-ante stage and motivating appropriate efforts at the interim stage. These dual objectives are embedded in an optimal share contract solution: a pair of startup funds and output share between the principal and agent. The target mechanism's performance is then assessed with respect to implementation of the optimal share contract solution. The focus is on linear contracts for their *applicability* and *practical relevance*.

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1 Introduction

A standard procedure in many organizations involves employees submitting performance targets. For capital investment projects, managers are required to make projections of revenue streams. Within firms, departments competing for higher budgets may make projections of additional revenue generation. Semi-governmental credit cooperatives may give subsidized credit to individuals for projects without collateral or any history of past performance, but they may still try to unravel any project's true potential by requiring varied repayment terms based on requested amount of loans and actual performance. Small-to-medium size entrepreneurs are required to submit business proposals for funding from banks or venture capitalists or business angels. In share tenancy in agriculture, a tenant may indicate anticipated crop produce for an absentee landlord's decision regarding the supply of fertilizers or other forms of capital support. Public and private organizations (for example, universities) often require researchers to specify intended outcomes to secure funding for R&D projects.

In all the above applications, the principal (employer, lender or landlord) commits valuable resources into a project whose productivity depends on both the quality of the project (same as the agent's type in our formulation) and the effort of the agent managing the resources. If the project's (or agent's) quality cannot be observed and the agent's effort cannot be monitored, the final output offers only a crude performance measure solely on which to base the incentives. The question of resources (capital/funding/loans as discussed above) is an *ex-ante* decision, so project screening is essential. Simultaneously, inducing the agent to exert effort at the *interim* stage (following resource provision) is a standard incentive problem. How can these dual objectives be aligned? With this being the primary concern, we analyze the performance of a target-based incentive mechanism.

In terms of analysis, we focus on what we consider to be a plausible implementation objective for the principal: the optimal share contract, which belongs to the special but important class of contracts, *affine/linear contracts*. The optimal share contract solution is defined in terms of the resources the agent should receive and a split of output between the principal and the agent, depending on project's type. We also derive the unrestricted second-best contract. However, our focus on linear, rather than optimal, contracts is motivated by the former's *practical relevance* and *easy applicability*. See Murphy (1999), Prendergast (1999), Lafontaine and Slade (2000) and Chiappori and Salanie (2003) for evidence on the prevalence of fairly simple contracts in the real world.

The core of our analysis concerns implementation of the optimal share contract solution using target incentives that are *piecewise-linear* in output and the shortfall of output from the agent's projected target.[1] We ask whether such a mechanism can efficiently screen a project's type allowing the principal to channel initial capital according to the project's potential. The mechanism we employ involves only a "small" reduction in the agent's reward for a "small shortfall". It is well-known in the mechanism design/agency literature that, with large penalties, agent misbehavior can be controlled but these may not be very practical solutions. Achieving

[1] McAfee and McMillan (1986) and Laffont and Tirole (1986) studied similar linear contracts for incentive provision.

desirable agent behavior with continuous (rather than large, discrete) adjustment in incentives should be part of an ideal mechanism design objective. Moreover, although issues of *fairness* and *inequity aversion* cannot be captured in our model, human attitude toward punishment tends to be of a compassionate nature; see, among others, Charness (2004) and Fehr et al. (2007) in the context of experiments involving moral hazard problems. Taking a similar view, any reasonable suggestion for performance improvement must not push the accountability doctrine too far; a proportionate reduction in salary for underperformance is going to be less contentious. While we focus on proportionate reward adjustments, we also demonstrate the potency of target mechanisms when discrete/lump-sum adjustments are feasible.

Our findings are as follows. When output is deterministic, using the target mechanism, there is proper disbursement of resources and the optimal share contract is implemented (Proposition 1). In the stochastic case, the high-ability agent can always be induced to announce a high target and produce output equal to the high type's optimal share contract level with observable types (Proposition 4). Whether the low-ability agent (truthfully) declares a low target or high target might depend on the random component of output. For large random components, a target contract may not always induce a low-type agent's self-selection of targets. However, if the random component is sufficiently small on average, the target contract ensures costless resolution of the adverse selection problem along with channeling of resources according to agent types (Proposition 5). Further, if we allow for discrete adjustments in the agent's reward (to be made clear later in the paper), the target contract performs reasonably well even for non-negligible random output component (Proposition 6). Overall, our results suggest a positive role of targets for screening of agent types and more efficient disbursement of resources, even beyond the class of linear contracts.

In general, resource provision by the principal implies that there is more on the line for the principal due to the resource cost. The agent tries to manoeuver the principal towards providing higher resources, who responds by using a richer contract space – output and resource contingent reward. This gives more leverage to the principal. Overall, the principal faces a more challenging task – to steer the agent towards appropriate resource support *and* effort. One might expect that the principal's share would increase with the resources she provides. However, overcoming the information asymmetry about agent/project type requires the principle to accept output shares that do not necessarily increase (and may possibly decrease) with the resources she provides. This is essentially due to the information rent that accrues to the high-ability agent who is provided higher resource support. It should be of interest that the target mechanism, using small proportionate reward adjustments, can properly disburse resources and overcome adverse selection and moral hazard problems.

In the next section, we discuss the related literature. Section 3 presents the optimal share contract solution for a deterministic production technology. In section 4, we introduce the target contract. Section 5 generalizes the target contract to the case of production uncertainty. Section 6 concludes. Most proofs are relegated to the Appendix.

2 Related literature

Bolton and Dewatripont (2005) provides an overview of the issues explored in the contract theory literature, which is too vast to be adequately reviewed here. We discuss closely related papers here. Using the principal-agent framework, the contract theory literature has earlier explored the role of targets in incentive provision in the presence of both adverse selection and moral hazard (for instance, Laffont and Tirole (1986), McAfee and McMillan (1986), Zou (1992)). McAfee and McMillan (1986) consider a cost overrun linear contract in a model of bidding for government contracts. If the realized cost exceeds the winning firm's bid, the firm is responsible for a fraction of the cost overrun, while if the cost is below the bid the firm keeps a part of the cost savings. Laffont and Tirole (1986) also studied a similar incentive contract involving cost overruns. In their model, a supplier has private information about a project's cost and can invest in cost reduction. The supplier announces expected costs and the optimal contract is linear in cost overruns. Zou (1992) studies a principal-agent production setting with moral hazard and the agent's 'type' affecting the disutility of effort. He shows that a threat-based incentive mechanism allowing the agent to select the levels of an exogenous lump-sum penalty and a corresponding minimal target output would eliminate moral hazard but the principal must concede information rent to the agent. The role of targets in pure moral hazard settings has been analyzed by Liu (1986) and Osband (1987). They find that if *extremely severe* penalties are imposed by the principal for output falling below a threshold target level, the penalty is almost never imposed ex-post.

Among recent works, Gershkov and Perry (2012) consider a dynamic principal-agent model with both moral hazard and adverse selection where an agent, whose type (high or low) is not known to the principal faces a sequence of tasks that are either easy or difficult (known only to the agent). To induce only the high quality agent to take up the difficult task, the principal should reward the agent only if the agent succeeds over a minimal number of successive tasks. The dynamic structure helps the principal to match high-quality agents with difficult tasks. The minimal sustained performance requirement in Gershkov and Perry (2012) is similar to the trigger of a target performance in our static setting.

What is missing from the above literature (both the static contracting literature and the dynamic version as in Gershkov and Perry (2012)) is an analysis of resource provision by the principal. In the applications discussed in the Introduction, the principal needs to supply the "seed" fund for a project to be viable and would like to choose an amount according to the project's merit.

Empirical evidence suggests that contracts used in practice are often *not* the optimal contracts predicted in theory – see, for instance, Murphy (1999), Prendergast (1999), and Lafontaine and Slade (2000). Further, Hart and Holmstrom (1987) and Chiappori and Salanie (2003) highlight the prevalence of fairly simple contracts in the real world. More importantly, share contracts come as natural consideration in *partnerships*, and are frequently observed in corporate and agricultural settings. For instance, *sales contracts* and *sharecropping* in agriculture are usually on a percentage basis. In agriculture, Laffont and Matoussi (1995) have highlighted the importance of share contract among the three main categories of contracts (rental contract and wage contract being the other two); for other studies see Bardhan and Srinivasan (1971), Hallagan (1978), Braverman and Stiglitz (1982), Allen (1982), Basu (1992), and more recently de Janvry and Sadoulet (2007) and Sen (2011). Executive pay packages often include *stock options* conditional on a threshold performance (Murphy (1999)). *Profit sharing* or *equity participation* is also considered to be a reasonable way to align employer-employee objectives (Weitzman (1985), Blanchflower and Oswald (1987), Grout (1988), Raith (2008)). These papers indicate the practical relevance and wide applicability of share contracts; hence, our focus on this class of linear contracts.

3 The model

A principal hires an agent to produce output that is a function of the agent's effort, the agent's (or the specific project's) type (productivity) and some key input (resource) the principal supplies. A contract between the principal and the agent is a sharing rule defined in terms of output and a *target* of achievement chosen by the agent. The principal does not observe the agent's effort (*moral hazard* problem) nor knows the agent's productivity (*adverse selection* problem). How much resource the principal allocates to the agent to facilitate production should ideally depend on the agent's type, but due to lack of information about the agent's type, the principal may allocate resources that are either too high or too low (*resource allocation* problem). The use of targets is primarily intended to solve this information problem.

The production function is of the Cobb-Douglas family:

$$y = \omega^\eta \alpha_\tau e, \qquad \text{with } \eta \in (0, \frac{1}{2}), \tag{1}$$

where ω is the resource provided by the principal for the project, α_τ is the agent's marginal productivity parameter, and e is the agent's effort. The restriction $\eta < 1/2$ is to make the resource allocation problem interesting. Later, we will see that if $\eta \geq 1/2$, there will be non-decreasing returns to resources and the principal would like to supply an unbounded amount of resources to the agent irrespective of the agent's type; see, also, footnote 6. In that case, the resource allocation is determined by the limit on the resources that the principal has access to. Then, the problems of moral hazard and adverse selection take a back seat. We assume that the resource given is specific to the project and cannot be "stolen" by the agent for personal consumption. The agent is one of two possible types, $\tau \in \{h, \ell\}$, with $\alpha_\tau \in \{\alpha_\ell, \alpha_h\}$, where $0 < \alpha_\ell < \alpha_h$; $\alpha_\tau = \alpha_h$ (i.e., the agent is of a 'high' type) with probability p and $\alpha_\tau = \alpha_\ell$ (the agent is a 'low' type) with probability $1 - p$.

The agent's utility is separable in income and effort, $U = R(y) - \frac{1}{2}e^2$, where $R(y)$ is the compensation from the principal. As is standard, we assume that the cost of effort is increasing and strictly convex.[2] The agent is risk-neutral in income and faces *limited liability*; the latter requires $R(y) \geq 0$, $\forall y \geq 0$. The principal is risk-neutral and her payoff is $V = y - R(y) - \omega$.[3]

[2] Similar quadratic effort cost formulations are used in Osband (1987), Prendergast (1999) and Raith (2008), among others.

[3] Note that all accounting is done in equivalent real terms; see, for instance, Raith (2008).

Lemma 1. *The **first-best** outcome involves the following type-contingent*

(*i*) *resources:* $\omega_{fb,\tau} = (\eta\alpha_\tau^2)^{\frac{1}{1-2\eta}}$,

(*ii*) *agent effort:* $e_{fb,\tau} = (\alpha_\tau\eta^\eta)^{\frac{1}{1-2\eta}}$, *and*

(*iii*) *output:* $y_{fb,\tau} = (\alpha_\tau\eta^\eta)^{\frac{2}{1-2\eta}} = e_{fb,\tau}^2$.

The subscript fb denotes the first-best solutions. The proof appears in the Appendix.

We are going to focus on general affine contracts of the form $R(y) = T + \delta y$ where $T \gtreqless 0$. One can consider setting $T < 0$ so that the agent makes a fixed transfer to the principal – a surplus extraction device. If, in addition, $\delta = 0$, this would imply the principal selling the project to the risk-neutral agent. However, given that the principal has to finance the project in the first place, asking for a fixed positive payment from the agent when there might even be zero output would be a contradiction. Also, such contracts cannot be implemented due to the limited liability constraint. On the other hand, setting $T > 0$ is not necessary in our setup as any value the principal can obtain with this contractual form under full information can be improved upon using a pure share contract. Hence, we restrict our attention to $T = 0$ and our emphasis will be on *share contracts* of the form:

$$R(y) = \delta y, \qquad 0 < \delta < 1. \tag{2}$$

Share contracts, part of a family of affine contracts, are simple and widely practised in various production settings as discussed in section 1.

Optimal share contract. We will propose an incentive mechanism to tackle the *adverse selection* problem. Specifically, we set out our objective to implement the *optimal share contract*, i.e., the share contract that maximizes the principal's payoff under the "as if" presumption that the *principal knows the agent's type*. In this context it may be noted that, even if the principal may not directly observe the agent's effort, for deterministic production as in (1), the effort exerted by the agent can be precisely inferred from the output and, thus, moral hazard is not really an issue.[4] It is straightforward to construct a mechanism implementing first-best (type contingent) effort levels and extracting full surplus for the principal.[5]

Given resources, ω, and a share contract as specified in (2), the agent solves:

$$\max_{e \geq 0} \quad U = \delta\omega^\eta\alpha_\tau e - \frac{1}{2}e^2,$$

to determine her effort level (that depends on the agent's type and the resources provided by the principal) as follows (note that $\frac{\partial^2 U}{\partial e^2} = -1 < 0$):

$$e_\tau = \delta\omega^\eta\alpha_\tau, \qquad \tau = \ell, h. \tag{3}$$

[4] Laffont and Martimort (2002) call this is a 'false moral hazard' problem.

[5] In fact this is true even when there could be noise due to the presence of a stochastic term in production so long as there exists output levels that reveal with probability arbitrarily close to 1 that the agent defected from a prescribed effort (see *Remark 1* in section B1.3 of Ch. 1 of Laffont and Tirole (1993)). However, the penalty for deviations will have to be lump-sum.

The principal chooses δ and ω to maximize her surplus, $V = (1-\delta)y - \omega$, given the agent's optimal effort choice. Using e_τ in (3), rewrite the principal's problem as:

$$\max_{\delta,\omega} \;\; V = \delta(1-\delta)\alpha_\tau^2\omega^{2\eta} - \omega,$$

solving which, using the first-order conditions, yields:[6]

$$\omega_{sc,\tau} \;=\; \left[2\eta\delta_{sc}(1-\delta_{sc})\alpha_\tau^2\right]^{\frac{1}{1-2\eta}} \tag{4}$$

$$\text{and,} \qquad \delta_{sc,\tau} \;=\; \delta_{sc} \;=\; \frac{1}{2}, \tag{5}$$

where subscript, *sc*, refers to optimal share contract. This leads to:

$$y_{sc,\tau} = \delta_{sc}(\omega_{sc,\tau})^{2\eta}\alpha_\tau^2, \qquad \tau = \ell, h. \tag{6}$$

We refer to $(\omega_{sc,\tau}, \delta_{sc,\tau})$ as the *optimal share contract solution*, given observable agent types. We are interested in using target based incentives to implement this solution when the agent's type is unknown to the principal.

4 Target announcements under adverse selection

Henceforth, we assume that the agent's type is unknown to the principal and analyze how target projections can resolve this information asymmetry. We consider a *modification* to the share contract: suppose that before allocating resources (ω), the principal requires the agent to announce a *target* output, $\hat{y}$. Then, given the resources (that depend on the projected target), the actual output and the announced target, the agent's reward is determined as follows:

[Target contract]

$$\begin{aligned}
&\text{If } \hat{y} < y_{sc,h}: \quad \omega = \omega_{sc,\ell}, \quad R = \delta_{sc}y \;; \\
&\text{If } \hat{y} \geq y_{sc,h}: \quad \omega = \omega_{sc,h}, \quad R = \begin{cases} \delta_{sc}y & \text{if } y \geq y_{sc,h}\;, \\ \delta_{sc}y - \underbrace{\min\{(1-\delta_{sc})[y_{sc,h} - y], \delta_{sc}y\}}_{\text{reward adjustment}} & \text{if } y < y_{sc,h}\;; \end{cases}
\end{aligned} \tag{7}$$

where y is the realized output, and $y_{sc,h}$ is the output under the optimal share contract for the high-type agent as determined in (6). ||

It is worth elaborating on the nature of the contract. The principal, in keeping with the optimal share contract solution with known types, offers a more generous resource support, $\omega_{sc,h}$, when the projected target is high, at least $y_{sc,h}$, and a moderate support for targets below

[6] The second-order conditions are also satisfied: $\frac{\partial^2 V}{\partial \omega^2} = 2\eta(2\eta-1)\delta(1-\delta)\alpha_\tau^2\omega^{2\eta-2} < 0$, if (and only if) $\eta \in (0, \frac{1}{2})$; $\frac{\partial^2 V}{\partial \delta^2} = -2\alpha_\tau^2\omega^{2\eta} < 0$; $\frac{\partial^2 V}{\partial \omega^2}\frac{\partial^2 V}{\partial \delta^2} - \left(\frac{\partial^2 V}{\partial \omega \partial \delta}\right)^2 = \eta(1-2\eta)\alpha_\tau^4\omega_{sc,\tau}^{4\eta-2} > 0$ (using $\delta_{sc} = \frac{1}{2}$).

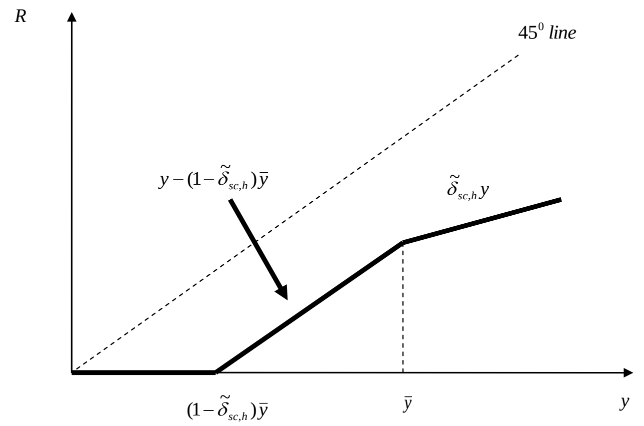

Fig. 1a: Agent's share with projection of a high target

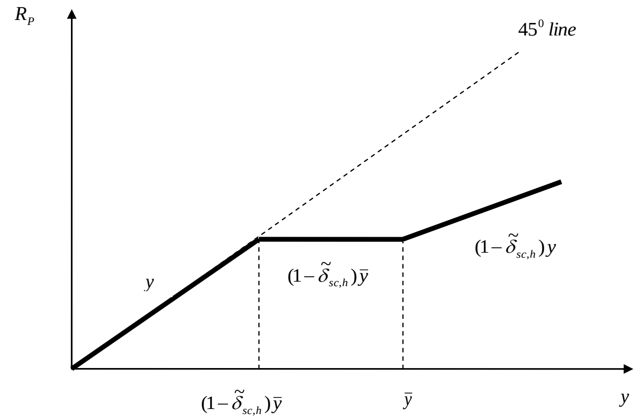

Fig. 1b: Principal's share when agent projects high target

this threshold.[7] Without the target announcement, a linear contract cannot separate between agent types and, thus, the principal has to offer a pooling contract by awarding resources for an average-type agent, which would result in a lower surplus for the principal.[8] Also, for proper disbursement of the more generous support, the principal adjusts the agent's reward if the latter underperforms after announcing a high target. Note, however, that this adjustment is kept within *reasonable bounds*. First, the agent is not asked to compensate the principal in excess of the amount of the loss in the latter's share due to underperformance. And, even then, the agent is never asked for compensation that is more than what would have been her share of output, $\delta_{sc}y$, thus satisfying the *limited liability* constraint. That is, when the agent announces a high target and fails to meet the target, her status is one of residual claimant. In this case, the agent's reward in (7) can be written as $R = \max\{0, y - (1 - \delta_{sc})y_{sc,h}\}$, which implies that, for output realization lower than $y_{sc,h}$, the agent receives a cut in her share, while for output realization higher than $y_{sc,h}$, the optimal shares are respected. This prioritization is to prevent the agent from projecting excessive targets to receive high resource support from the principal. The contract is presented in Figs. 1a-b. These figures are also going to be applicable to our analysis in section 4. For now, one should treat $\tilde{\delta}_{sc,h} = \delta_{sc}$ and $\bar{y} = y_{sc,h}$.

Relevance of target contracts. For practical relevance of target incentives, Fig. 1a

[7] Alternatively, the principal could require the agent to announce resource support needed. Of course, there would be appropriate changes to the contract offered: for instance, if an agent asks for resources $\hat{\omega} \geq \omega_{sc,h}$, she would be required to produce output $y \geq y_{sc,h}$ to avoid reward adjustment.

[8] Such a pooling contract is presented in the Appendix.

can be compared to Fig. 5 in Murphy (1999) (reproduced as Fig. 4 at the beginning of the Appendix). He finds that almost all companies have explicit annual bonus plans based on single-year performances with some covering only CEOs, while others include all employees. The typical plan pays a bonus (in percentage terms) conditional on a threshold performance and specifies a maximum bonus. In the incentive mechanism we have considered, although there is no exact parallel of bonus, for high target projection the marginal incentive at the interim stage is at its maximum between output level $(1 - \tilde{\delta}_{sc,h})\bar{y}$ and the target output $\bar{y}$ as the agent is the residual claimant.

The empirical findings of Murphy (1999) suggest that bonus contracts are primarily aimed at inducing agents towards better performance at the interim stage, i.e., solving the moral hazard problem. Our task involves an extra challenge: not only must the agent be incentivized to exert effort at the *interim* stage, resources must also be allocated correctly at the *ex-ante* stage. We have, therefore, chosen the specific incentives in (7), and later on in (14) in section 4, after incorporating the insights from the earlier theoretical literature (e.g., Laffont and Tirole (1986) and McAfee and McMillan (1986)), to evaluate the mechanisms' performance in mitigating information asymmetry problems at both the ex-ante stage (about project type) and the interim stage (about project type and efforts). Simultaneously, the mechanism's reward adjustment for underperformance is kept within reasonable bounds.

Analysis of target contract. Incentive compatibility under the target contract (7) requires: $U_\ell(\hat{y} < y_{sc,h}) \geq U_\ell(\hat{y} \geq y_{sc,h})$ and $U_h(\hat{y} \geq y_{sc,h}) \geq U_h(\hat{y} < y_{sc,h})$. That is, the low-type should be better off to announce a target below the threshold, while the high-type should be better off by announcing a target at least $y_{sc,h}$.

Assuming that the agent has zero outside option, both types must find it optimal to participate (ex-ante individual rationality constraint):

$$U_\ell(\hat{y} < y_{sc,h}) \geq 0, \quad U_h(\hat{y} \geq y_{sc,h}) \geq 0.$$

If an agent of type τ announces a target $\hat{y} < y_{sc,h}$, she is provided with the resources $\omega = \omega_{sc,\ell}$ and her problem is:

$$\max_e \ \delta_{sc}(\omega_{sc,\ell})^\eta \alpha_\tau e - \frac{1}{2}e^2,$$

which yields the equilibrium effort: $e^*_\tau = \delta_{sc}(\omega_{sc,\ell})^\eta \alpha_\tau, \qquad \tau = \ell, h.$

If an agent of type τ announces a target $\hat{y} \geq y_{sc,h}$, she is provided with resources $\omega = \omega_{sc,h}$. Her problem is then whether to produce at least $y_{sc,h}$ and not incur the downward reward adjustment for underperformance, or produce less than $y_{sc,h}$ and receive the lower compensation.

Our first claim is that a high-type agent would announce a target at least $y_{sc,h}$ and deliver $y_{sc,h}$. Note that, having announced $\hat{y} \geq y_{sc,h}$, a high-type agent will not underperform: by putting in her optimal effort $e_h = \delta_{sc}(\omega_{sc,h})^\eta \alpha_h$ (as given in (3)) and, thus, producing $y_{sc,h}$, the agent receives a strictly higher (net) utility than if she had put in a lower effort with the same resource support, $\omega_{sc,h}$, under the optimal share contract (of section 2) but *without* the provision of reward adjustment for underperformance specified in (7). So, under the target contract, (7), to exert lower effort than the optimal share contract effort, $e_h = \delta_{sc}(\omega_{sc,h})^{\eta}\alpha_h$, and incur the additional cost of downward reward adjustment (i.e., a lower share of output) can only further lower the agent's net utility (as her compensation function under (7) is otherwise the same as in the optimal share contract), which cannot be optimal. It is easy to check that the agent will not produce higher than $y_{sc,h}$. It now remains to show that the high-type agent will not project $\hat{y} < y_{sc,h}$. By projecting $\hat{y} < y_{sc,h}$, the high-type agent would receive $U_h(\hat{y} < y_{sc,h}) = (1/2)\delta_{sc}^2(\omega_{sc,\ell})^{2\eta}\alpha_h^2$, whereas $U_h(\hat{y} \geq y_{sc,h}) = (1/2)\delta_{sc}^2(\omega_{sc,h})^{2\eta}\alpha_h^2$. Since $\omega_{sc,h} > \omega_{sc,\ell}$, projecting $\hat{y} \geq y_{sc,h}$ strictly dominates.

As for the low-type agent's incentives, either she projects as low type (i.e., $\hat{y} < y_{sc,h}$) and receives a strictly positive payoff (as in the optimal share contract) or projects as high type (i.e., $\hat{y} \geq y_{sc,h}$). In the latter case, suppose she exerts effort, $\acute{e}$, that leads to an output less than $y_{sc,h}$.[9] Then, there are two possibilities: (i) the agent's compensation is adjusted such that she forgoes all her salary income due to the reward adjustment, and (ii) the agent's post-adjustment share still leaves her with a strictly positive payoff. In case (i), the agent receives zero payoff and this is clearly dominated by the strategy of truthful target projection as the low type, i.e., the self-selection constraint rules out this outcome. In case (ii), the principal receives $(1-\delta_{sc})y_{sc,h}-\omega_{sc,h}$ which is *higher* than her optimal share contract payoff, $(1-\delta_{sc})y_{sc,\ell}-\omega_{sc,\ell}$, for the ℓ-type agent. Thus, for the principal, the implemented outcome under the target contract, (7), is at least as good as the outcome under the optimal share contract (with known types), and possibly better.[10]

The main message of our argument in this section can now be formally stated:

Proposition 1 (**Optimal share contract implementation using targets**). *A menu of piece-wise linear contracts, specified by* (7)*, specifying* ***target outputs*** *is able to implement resource allocation and efforts that are at least as good as those in the optimal share contract (with known types). The target contract has the following features:*

(i) for target announcement $\hat{y} < y_{sc,h}$, optimal resource for the low-type agent, $\omega_{sc,\ell}$, is awarded along with low-powered incentives – the agent receives a fixed share, δ_{sc}, of output;

(ii) for target announcement $\hat{y} \geq y_{sc,h}$, more generous, optimal resource for the high-type agent, $\omega_{sc,h}$, is awarded along with high-powered incentives – the agent receives a fixed share, δ_{sc}, of output if she meets the target; if there is a shortfall from the projection, her compensation is lowered proportional to the shortfall in actual output from the projection.

Remark 1. *The agent's effort cost in the first-best case is half the output, which implies equal shares for the principal and the agent. This is similar to the share implemented by the target contract.*

[9] If the agent produces $y_{sc,h}$ or more, the principal gains relative to the optimal share contract.

[10] It is worth pointing out how the target contract, (7), might even achieve strictly more than the optimal share contract payoff for the principal. While the target contract is (piece-wise) linear in output, the reward adjustment scheme (which makes the agent residual claimant) adds a negative component to the agent's compensation (which is a fraction of the projected target) by which the principal can dip into part of the (first-best) surplus that may have been lost due to our restriction to share contract solution as the implementation objective.

Due to unknown agent types, the principal may not be able to channel key resources commensurate with agent types. Asking the agent to choose from a menu of target based contracts that adjust the agent's share for not being able to meet her chosen threshold target level resolves the adverse selection problem. What should be of interest is the effectiveness of the *proportionate* reward adjustment scheme. Our mechanism prescribes only occasional (downward) reward adjustment in the form of lower output share and no bonus; further, when the reward is adjusted, it is proportional to the extent of underperformance. The mechanism can be contrasted with the unrestricted second-best contract (which we present next) and Zou (1992) where the optimal threat-based incentive mechanism employs a lump-sum penalty that is independent of the degree of shortfall in output – see section 4.1 of Zou's paper.

Unrestricted second-best contract. Suppose the principal offers a menu of contracts, $\{\omega_\tau, y_\tau, R_\tau\}_{\tau\in\{\ell,h\}}$ contingent on the type, τ, reported by the agent; that is, the agent is given resources ω_τ, has to produce output y_τ and her compensation is R_τ. Denote the unrestricted second-best contract solutions using the subscript *ur.sb*.

Proposition 2. *The unrestricted second-best contract has the following features:*

(*i*) *the first-best outcome cannot be achieved;*

(*ii*) *the type-contingent resource allocations are:*

$$\omega_{ur.sb,h} = (\eta\alpha_h^2)^{\frac{1}{1-2\eta}} = \omega_{fb,h}, \quad \text{and} \quad \omega_{ur.sb,\ell} = (\gamma\eta\alpha_\ell^2)^{\frac{1}{1-2\eta}} < \omega_{fb,\ell}, \qquad \gamma \equiv \frac{1-p}{1-p\frac{\alpha_\ell^2}{\alpha_h^2}} < 1;$$

(*iii*) *the output produced by each type are:*

$$y_{ur.sb,h} = (\alpha_h\eta^\eta)^{\frac{2}{1-2\eta}} = y_{fb,h}, \quad \text{and} \quad y_{ur.sb,\ell} = \gamma^{\frac{1}{1-2\eta}}(\alpha_\ell\eta^\eta)^{\frac{2}{1-2\eta}} < y_{fb,\ell};$$

(*iv*) *the agents' efforts are:*

$$e_{ur.sb,h} = (\alpha_h\eta^\eta)^{\frac{1}{1-2\eta}} = e_{fb,h}, \quad \text{and} \quad e_{ur.sb,\ell} = \gamma^{\frac{1-\eta}{1-2\eta}}(\alpha_\ell\eta^\eta)^{\frac{1}{1-2\eta}} < e_{fb,\ell};$$

(*v*) *the agents' compensations are:*

$$R_{ur.sb,h} = \frac{1}{2}\left(\frac{y_{ur.sb,h}}{\omega_{ur.sb,h}^\eta\alpha_h}\right)^2 + \underbrace{\frac{1}{2}\left(\frac{y_{ur.sb,\ell}}{\omega_{ur.sb,\ell}^\eta\alpha_\ell}\right)^2 - \frac{1}{2}\left(\frac{y_{ur.sb,\ell}}{\omega_{ur.sb,\ell}^\eta\alpha_h}\right)^2}_{\textit{high-type's information rent } > \, 0}, \text{ and } R_{ur.sb,\ell} = \frac{1}{2}\left(\frac{y_{ur.sb,\ell}}{\omega_{ur.sb,\ell}^\eta\alpha_\ell}\right)^2.$$

The proof is relegated to the Appendix. Under the unrestricted second-best contract, the low-type agent is compensated her effort cost, while the high-type receives an additional information rent. The high-type agent is provided the first-best resources, incentivized to exert her first-best effort (although at a higher cost to the principal), and produce the first-best output. The low-type agent's resource support, effort and output are lower than those in the first-best case. Numerical comparisons presented in the Appendix show that the optimal share

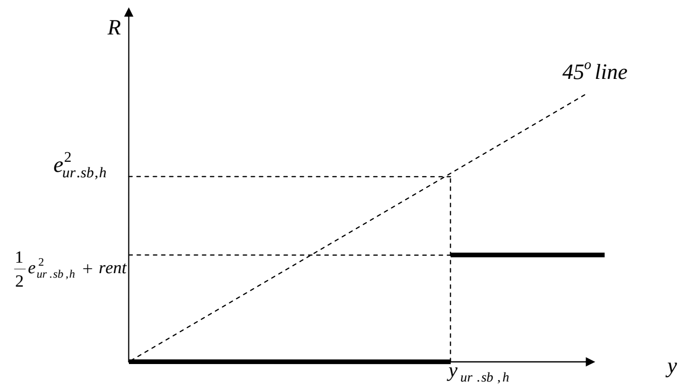

Fig. 2a: Agent's remuneration with high target projection

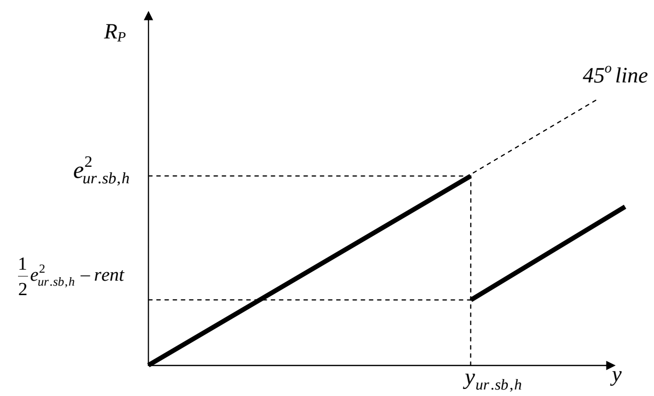

Fig. 2b: Principal's remuneration when agent projects high target

contract, as expected, results in a welfare loss for the principal as compared to the unrestricted second-best contract.

Although we focus on implementation of the optimal share contract for reasons discussed in section 1, the target mechanism can also be used to implement the second-best contract:

Remark 2 (**Second-best contract implementation using target mechanism**). *Suppose the principal requires the agent to announce a* ***target output****, $\hat{y} \in \{y_{ur.sb,\ell}, y_{ur.sb,h}\}$. Then, resources are allocated and agent remuneration determined as follows:*

$$
\begin{aligned}
&\textit{If } \hat{y} = y_{ur.sb,\ell}: \quad \omega = \omega_{ur.sb,\ell}, \qquad R \;=\; \begin{cases} R_{ur.sb,\ell} & \textit{if } y \geq y_{ur.sb,\ell} \\ 0 & \textit{if } y < y_{ur.sb,\ell} \end{cases} \\
&\textit{If } \hat{y} = y_{ur.sb,h}: \quad \omega = \omega_{ur.sb,h}, \qquad R \;=\; \begin{cases} R_{ur.sb,h} & \textit{if } y \geq y_{ur.sb,h} \\ 0 & \textit{if } y < y_{ur.sb,h} \end{cases}
\end{aligned}
$$

where y is the realized output, while $\omega_{ur.sb,\tau}$, $y_{ur.sb,\tau}$ and $R_{ur.sb,\tau}$ are the resources, output and remuneration for τ-type agent under the optimal contract. This target contract is able to implement the unrestricted second-best contract outcomes of Proposition 2.

The unrestricted second-best contract is represented in Figs. 2a-b. The agent's compensation is lowered to **zero** if she misses the prescribed output, something we rarely see in practice. This is also evident if we compare Fig. 2a to Fig. 5 in Murphy (1999) which depicts contracts used in practice (reproduced at the beginning of the Appendix). Note that the horizontal segment in Murphy prior to the threshold performance is the bonus, *not* total compensation (as in

Fig. 2a). Further, as discussed in section 1, it is well known that optimal contracts predicted by theory are seldom used in firms. Hence, in the rest of the paper, we mainly focus on the target contract of the form in (7), with proportionate reward adjustment for two reasons – *practical relevance* and *wide applicability.*

5 Stochastic output and the role of targets

We now extend our analysis to the case of production uncertainty; output is now subject to a random shock, ϵ:

$$y = \omega^{\eta}\alpha_{\tau}e + \epsilon, \qquad \text{with } \eta \in (0, \frac{1}{2}), \tag{8}$$

where ϵ has a continuously differentiable distribution, $F(\epsilon)$, with density $f(\epsilon) > 0$ on the support $[0, \epsilon^{+}]$, and $F(0) = 0$; $F(.)$ is common knowledge and $\mathbb{E}(\epsilon) = \bar{\epsilon}$, where $\mathbb{E}$ is the expectation operator.[11]

At this stage we would like to emphasize that we will be considering technologies with *small* random shocks, that is, $\bar{\epsilon}$ not too large. If there is a large element of chance (as would be the case with high $\bar{\epsilon}$), then screening the employee's type or motivating her to work hard using incentive schemes involving only small proportionate reward adjustment becomes difficult; a low-skilled employee may project high performance and take calculated risks since, on average, the random component of output would be high. So our analysis mostly focuses on small errors.

When the agent's effort is unobservable but her type is *known* to the principal, i.e., there is moral hazard but no adverse selection problem, for any share contract incentive (i.e., δ) set by the principal, the agent's problem is:

$$\max_{e \geq 0} \quad U = \mathbb{E}_{\epsilon}\left[\delta\left(\omega^{\eta}\alpha_{\tau}e + \epsilon\right)\right] - \frac{1}{2}e^{2},$$

which yields optimal effort as:

$$\tilde{e}(\omega, \delta) = \delta\omega^{\eta}\alpha_{\tau}, \qquad \tau = \ell, h. \tag{9}$$

The principal's problem can then be written as:

$$\max_{\omega,\delta} V = \mathbb{E}_{\epsilon}\left[\delta(1-\delta)\alpha_{\tau}^{2}\omega^{2\eta} + (1-\delta)\epsilon - \omega\right],$$

where we use the agent's optimal efforts from (9). Solving the first-order conditions,

$$\frac{\partial V}{\partial \omega} = 2\eta\delta(1-\delta)\alpha_{\tau}^{2}\omega^{2\eta-1} - 1 = 0, \tag{10}$$

$$\frac{\partial V}{\partial \delta} = (1-2\delta)\alpha_{\tau}^{2}\omega^{2\eta} - \bar{\epsilon} = 0, \tag{11}$$

yields the resources and shares under the ***optimal share contract*** with known types; we denote these as $(\tilde{\omega}_{sc,\tau}, \tilde{\delta}_{sc,\tau})$ (distinct from $(\omega_{sc,\tau}, \delta_{sc})$ in the deterministic case). That the solutions

[11] We choose a positive support for the random component of output; negative ϵ implies output will be negative if the agent chooses zero effort.

$(\tilde{\omega}_{sc,\tau}, \tilde{\delta}_{sc,\tau})$ exist and are interior (so that first-order conditions are appropriate) follow from the fact that V is continuous in (ω, δ), both δ and ω are chosen from compact sets (these together guaranteeing the principal's problem has a solution, by Weierstrass theorem),[12] and neither $\delta = 0$ or $\delta = 1$ can be a solution, nor can $\omega = 0$ or ω very large be the solution, for any agent type, τ. In the Appendix, we provide details regarding the verification of second-order conditions.

Using the solutions in (9), determine the optimal effort levels:

$$\tilde{e}_{sc,\tau} = \tilde{\delta}_{sc,\tau} \tilde{\omega}^{\eta}_{sc,\tau} \alpha_{\tau}, \qquad \tau = \ell, h. \tag{12}$$

Also, denote the deterministic component of output under the optimal share contract as:

$$\tilde{y}_{sc,\tau} = \tilde{\omega}^{\eta}_{sc,\tau} \alpha_{\tau} \tilde{e}_{sc,\tau}, \qquad \tau = \ell, h. \tag{13}$$

Proposition 3 (Monotonicity of optimal share contract solution with stochastic output). *When agent types are known, the optimal share contract solutions $\tilde{\omega}_{sc,\tau}$ and $\tilde{\delta}_{sc,\tau}$*

(i) are increasing in α_τ;

(ii) approach, respectively, the optimal share contract solutions in the non-stochastic environment, $\omega_{sc,\tau}$ and $\delta_{sc} = 1/2$ (given by (4) and (5)), as $\bar{\epsilon} \to 0$.

The proof is relegated to the Appendix. The optimal share contract, thus, offers higher initial resources and higher output share for the high-type agent. Next, we propose a target contract to deal with adverse selection and resource allocation problems under production uncertainty.

Target contract under uncertainty. Consider a target contract of the following form:

$$\begin{aligned}
&\text{If } \hat{y} < \bar{y}: \quad \omega = \tilde{\omega}_{sc,\ell}, \quad R = \tilde{\delta}_{sc,\ell}\, y \;; \\
&\text{If } \hat{y} \geq \bar{y}: \quad \omega = \tilde{\omega}_{sc,h}, \quad R = \begin{cases} \tilde{\delta}_{sc,h}\, y & \text{if } y \geq \bar{y}\,, \\ \tilde{\delta}_{sc,h}\, y - \underbrace{\min\{(1-\tilde{\delta}_{sc,h})[\bar{y} - y],\ \tilde{\delta}_{sc,h} y\}}_{\text{reward adjustment}} & \text{if } y < \bar{y}\,; \end{cases}
\end{aligned} \tag{14}$$

where $\bar{y}$ is the cut-off target set by the principal. ||

Remark 3. *Note that for only <u>two</u> types of agents (as in our setup), a single cutoff target, $\bar{y}$, is intended to separate between who gets more generous support and who gets moderate support. For more than two types, the target contract can be made further staggered by awarding smaller and smaller resources for lower announced targets corresponding to low-valued τ's.*

Also, note that, different from the target contract in (7) for the deterministic case, now the linear incentive, δ, depends on the agent's type ($\tilde{\delta}_{sc,\tau}$ as opposed to δ_{sc}). This additional flexibility is brought in to motivate the different agent types to self-select the contract meant

[12] Clearly ω cannot be unbounded, so compactness assumption is reasonable.

for her type. The higher the cut-off target, $\bar{y}$, the more difficult it becomes for either type to project the cut-off target.

Remark 4. *In general, one would expect that the principal's share would be increasing in the resources she provides. However, to overcome the adverse selection and moral hazard problems, the principal has to accept a lower share of output when she provides a higher resource support to the agent. This is the high-type agent's information rent.*

We now turn to an analysis of the agent's incentives and the following lemma, proved in the Appendix, establishes the agent's effort choice following a high target announcement.

Lemma 2 (Equilibrium effort following high target announcement). *Suppose $\bar{y} \geq \tilde{y}_{sc,h}$, and the agent has announced the cut-off target $\bar{y}$ to receive the more generous resource support. Then the equilibrium efforts, $e^*_{\tau,h}, \tau = \ell, h$, solving*

$$e^*_{\tau,h} = \tilde{\omega}^{\eta}_{sc,h}\alpha_\tau \left[\tilde{\delta}_{sc,h} + (1-\tilde{\delta}_{sc,h})F(\bar{y} - \tilde{\omega}^{\eta}_{sc,h}\alpha_\tau e^*_{\tau,h}) - F\left((1-\tilde{\delta}_{sc,h})\bar{y} - \tilde{\omega}^{\eta}_{sc,h}\alpha_\tau e^*_{\tau,h}\right)\right], \quad (15)$$

always exist. Moreover,

(*i*) *the equilibrium effort for each type is such that the deterministic part of output, $\tilde{\omega}^{\eta}_{sc,h}\alpha_\tau e^*_{\tau,h}$, is strictly less than the cut-off target, $\bar{y}$, when $\bar{y} > \tilde{y}_{sc,h}$;*

(*ii*) *if $\bar{y} = \tilde{y}_{sc,h}$, the high-type agent always meets the target (irrespective of the realization of ϵ) while the low-type agent may sometimes fail to meet the target.*

We can now present the first of three results for the uncertain output case.

Proposition 4 (High-type's self-selection under the target contract). *Suppose the principal sets the cut-off target $\bar{y} = \tilde{y}_{sc,h}$ and offers the target contract (14). The high-type agent self-selects by announcing a target $\hat{y} = \tilde{y}_{sc,h}$, exerts the optimal share contract effort level, $\tilde{e}_{sc,h}$, and <u>always</u> achieves the target.*

The proof (in the Appendix) follows the same line of reasoning as the arguments behind Proposition 1. While, for the target contract (14), the high-type agent self-selects and achieves the optimal share contract output level, the low-type agent's self-selection cannot always be guaranteed. The reason for this is not difficult to understand. If the random error term, ϵ, is large on average (relative to the deterministic part of the optimal share contract output for the high type), a low-type agent may take a chance on being lucky with the random error draw and announce a high target to take advantage of the more generous resource support and higher share of the output (recall, by Proposition 3, $\tilde{\omega}_{sc,h} > \tilde{\omega}_{sc,\ell}$ and $\tilde{\delta}_{sc,h} > \tilde{\delta}_{sc,\ell}$). As discussed earlier, there is a realistic prospect of making the target incentive contract deliver the desired separation of types at no (or insignificant) loss of efficiency as compared to the optimal share contract if the random error component is not too large. Further, as $\bar{\epsilon} \to 0$, the target contract under uncertainty becomes equivalent to target incentives without uncertainty. Hence, straightforward application of Propositions 1 and 3 yields:

Proposition 5 (**Limit result under uncertainty**). *Suppose the principal sets the cut-off target* $\bar{y} = \tilde{y}_{sc,h}$ *and offers the target contract* (14). *In the limit, as* $\bar{\epsilon} \to 0$, *the target contract under uncertainty,* (14), *becomes equivalent to the target contract in the deterministic case,* (7), *i.e.,* $\tilde{\delta}_{sc,\tau} \to \delta_{sc} = \frac{1}{2}$ *and* $\tilde{\omega}_{sc,\tau} \to \omega_{sc,\tau}$. *Furthermore, the target contract is able to achieve an outcome at least as good as that in the optimal share contract.*

Remark 5. *Note that, by continuity, for sufficiently small* ϵ, *the target contract can implement the optimal share contract outcome under uncertainty.*

Next, to demonstrate further the potential of the target contract, we allow more flexibility with regard to the reward adjustment scheme, whilst staying in the domain of (piece-wise) linear contracts. We modify the contract as follows: if output falls below the threshold target of $\bar{y} = \tilde{y}_{sc,h}$, the agent who announced the high target will receive only a fixed non-negative payment; otherwise her remuneration is the same as in the originally specified contract, (14). Also, we simplify by assuming that the random error, ϵ, is distributed *uniformly* over $[0, \epsilon^+]$. The modification comes at a cost, however, as the reward adjustment is no longer proportionate. Formally:

Target contract with discrete reward adjustment. Consider the following modification to the original target contract (specified in (14)):

$$\begin{aligned} &\text{If } \hat{y} < \bar{y}: \quad \omega = \tilde{\omega}_{sc,\ell}, \quad R = \tilde{\delta}_{sc,\ell}\, y \ ; \\ &\text{If } \hat{y} \geq \bar{y}: \quad \omega = \tilde{\omega}_{sc,h}, \quad R = \begin{cases} \tilde{\delta}_{sc,h}\, y & \text{if } y \geq \bar{y}\ , \\ \gamma\, y & \text{if } y < \bar{y}\ ; \end{cases} \end{aligned} \tag{16}$$

where $\bar{y}$ is the cut-off target set by the principal and $\gamma < \tilde{\delta}_{sc,h}$. ||

The contract (16), depicted in Figs. 3a-b (for the special case of $\gamma = 0$), may be contrasted with Figs. 1a-b representing our original target incentive contract. Note that, despite the discrete nature of reward adjustment, this variation of target incentives satisfies limited liability, and, thus, may be considered reasonable.

Proposition 6 (**Optimal share contract implementation with discrete reward adjustment**). *Suppose the principal offers the agent the target contract as specified in* (16), *sets* $\bar{y} = \tilde{y}_{sc,h}$, *and* $\gamma \to 0$. *Furthermore, assume that the random error,* ϵ, *follows a 'uniform' distribution and*

$$\epsilon^+ < \min\left\{\tilde{y}_{sc,\ell}\ ,\ \frac{\alpha_\ell^2}{(\alpha_h^2 - \alpha_\ell^2)}\tilde{y}_{sc,h}\right\}. \tag{17}$$

Then,

(*i*) *the high-type agent self-selects by announcing a high target and achieves the target;*

(*ii*) *the low-type agent either self-selects by announcing a low target or announces a high target and achieves the target;*

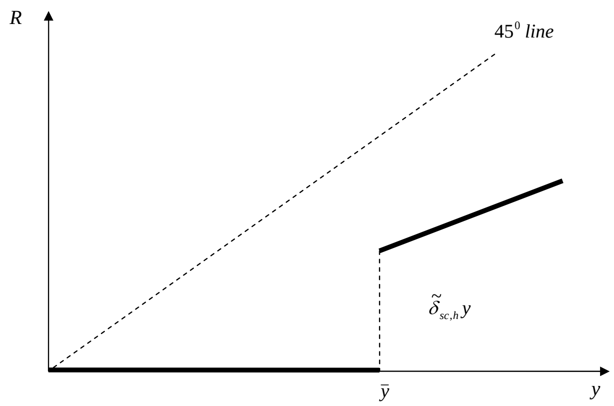

Fig 3a: Discrete reward adjustment with limited liability

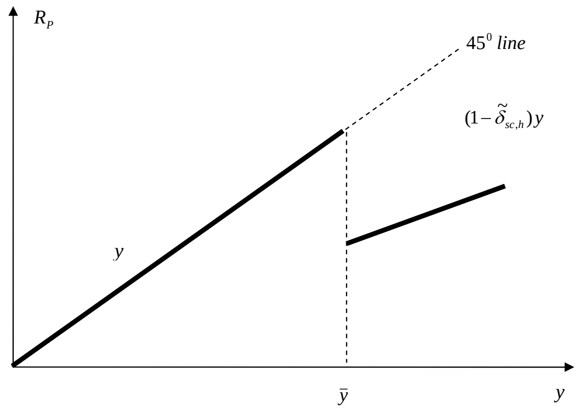

Fig 3b: Principal's share under discrete reward adjustment

(*iii*) *from the principal's perspective, the target contract specified in* (16) *achieves an outcome at least as good as the optimal share contract.*

Thus, for underperformance, making the reward adjustment discrete (as opposed to the earlier proportionate adjustment scheme), by significantly lowering the agent's share if she fails to produce her projected target, makes the target incentive contract more potent in screening agent types. This is true for errors that, on average, are bounded away from zero, so long as the average error is not too large relative to the output.

Propositions 5 and 6 confirm the merits of an important type of penalty mechanism[13] – the principle of lowering the agent's compensation for underperformance relative to her projected (or expected) level of performance. We show that *small and proportionate* adjustments (in the agent's compensation) based on targets can be useful for optimally awarding valuable scarce resources to agents who propose projects of uncertain quality. The earlier literature has *not* considered the resource allocation problem, which is important in many settings. Our analysis, thus, offers a further perspective on target incentives by looking at resource allocation as an explicit consideration for the principal.

[13] In a sense whether one uses the term *penalty* in describing our incentive mechanism, depends on what one considers to be the agent's rightful remuneration. Since the agent's share is prescribed in the contract to be low for below-target performance, one need *not* view this to be a penalty contract – just that the share is not as generous as it might have been if output had met the projected target.

6 Concluding remarks

Lack of accountability is often cited as one of the main reasons for non-delivery of performance in organizations. To make someone accountable means the person will have to bear the consequences of her actions. There are various ways incentives can be structured to make employees accountable. One common practice is that employees are required to make performance projections, i.e., announce targets, before being given the required resources to achieve the projected targets. To avoid exaggeration and wasteful spending of key resources, an employer may stipulate compensation adjustment clauses for non-delivery of the projected performance. This paper argues that for reward adjustments linear in the shortfall of the actual performance below the announced targets, i.e., adjustments that are "small" for "small underperformance", exaggeration of deliverables can be mitigated. This ensures that employees make realistic performance projections and organizations are thus able to better channel valuable resources according to each project's merit. Such contracts would be considered fair as compared to contracts that lower agent compensation significantly (possibly to zero) even for a small shortfall in output; the latter are seldom observed in practice (see, among others, Murphy (1999), Prendergast (1999), Charness (2004) and Fehr et al. (2007)).

Appendix

Components of a "typical" annual incentive plan

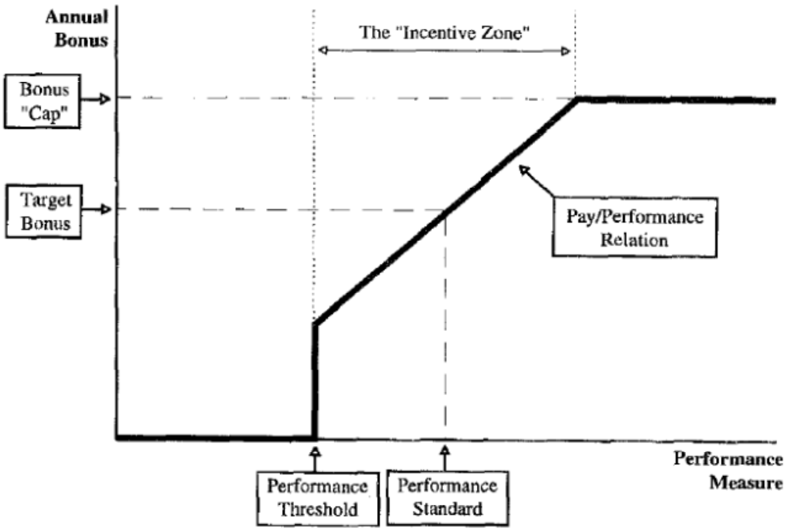

Fig 4: Reproduction of Figure 5 in Murphy (1999), p. 2499

Proof of Lemma 1. The first-best resource and effort allocations are given by the solution to:

$$\max_{\omega, e \geq 0} \quad W = \omega^{\eta} \alpha_{\tau} e - \frac{1}{2} e^2 - \omega.$$

Thus, the first-best resource and effort allocations, contingent on the agent's type, are:

$$\omega_{fb,\tau} = \left(\eta \alpha_{\tau}^2\right)^{\frac{1}{1-2\eta}}, \quad \text{and} \quad e_{fb,\tau} = \left(\alpha_{\tau} \eta^{\eta}\right)^{\frac{1}{1-2\eta}},$$

where the subscript fb refers to first-best. The first-best output level is $y_{fb,\tau} = (\alpha_{\tau}\eta^{\eta})^{\frac{2}{1-2\eta}}$.

The principal's surplus for each type is $V_{fb,\tau} = (\frac{1}{2} - \eta)(\alpha_{\tau}\eta^{\eta})^{(\frac{2}{1-2\eta})}$. **Q.E.D.**

Pooling share contract. When the agent's type is unknown, consider the share contract under both moral hazard and adverse selection problems. Given resources, ω, and a linear contract as specified in (2), the agent solves: $\max_{e \geq 0}\ U = \delta\omega^{\eta}\alpha_{\tau} e - \frac{1}{2}e^2$, which gives the agent's effort:[14] $e_{\tau} = \delta\omega^{\eta}\alpha_{\tau}$, $\tau = \ell, h$.

Using e_{τ} from above, the principal's problem can be written as: $\max_{\delta,\omega}\ \mathbb{E}_{\alpha}[V] = \mathbb{E}_{\alpha}[\delta\omega^{\eta}\alpha_{\tau}e] - \omega = \delta(1-\delta)\omega^{2\eta}\mathbb{E}_{\alpha}[\alpha_{\tau}^2] - \omega$, where $\mathbb{E}_{\alpha}[.]$ denotes the expectations operator over the agent's type. The first-order conditions for the principal's problem imply:[15]

$$\delta_P = \frac{1}{2}, \quad \text{and} \quad \omega_P = \left(2\eta\delta_P(1-\delta_P)\mathbb{E}_{\alpha}[\alpha_{\tau}^2]\right)^{\frac{1}{1-2\eta}} = \left[\frac{\eta}{2}\left(p\alpha_h^2 + (1-p)\alpha_{\ell}^2\right)\right]^{\frac{1}{1-2\eta}}, \tag{18}$$

where the subscript P denotes the pooling contract. The main difference between the pooling contract and the optimal share contract (implemented using the target mechanism) is in terms

[14] It is straightforward to check that $\frac{\partial^2 U}{\partial e^2} = -1 < 0$.

[15] The second-order conditions are satisfied: $\frac{\partial^2 \mathbb{E}_{\alpha}[V]}{\partial \omega^2} = 2\eta(2\eta-1)\delta(1-\delta)\omega^{2\eta-2}\mathbb{E}_{\alpha}[\alpha_{\tau}^2] < 0$, if $\eta \in (0, \frac{1}{2})$; $\frac{\partial^2 \mathbb{E}_{\alpha}[V]}{\partial \delta^2} = -2\omega^{2\eta}\mathbb{E}_{\alpha}[\alpha_{\tau}^2] < 0$.

of resources awarded to the agent – under the former, resources cannot be made contingent on the agent's type and this lack of flexibility means loss of surplus for the principal as compared to the optimal share contract that the target mechanism implements.

Proof of Proposition 2. The principal offers a menu of contracts$\{(\omega_\ell, y_\ell, R_\ell), (\omega_h, y_h, R_h)\}$ to induce self-selection by agents. If an agent reports type τ, she is given resource support ω_τ and compensated R_τ to produce output y_τ. The optimal contract is given by the solution to:

$$\max_{\{\omega_\tau, y_\tau, R_\tau\}_{\tau\in\{\ell,h\}}} \quad p(y_h - R_h - \omega_h) + (1-p)(y_\ell - R_\ell - \omega_\ell), \tag{19}$$

subject to

$$R_\ell - \frac{1}{2}\left(\frac{y_\ell}{\omega_\ell^\eta \alpha_\ell}\right)^2 \geq R_h - \frac{1}{2}\left(\frac{y_h}{\omega_h^\eta \alpha_\ell}\right)^2, \tag{20}$$

$$R_h - \frac{1}{2}\left(\frac{y_h}{\omega_h^\eta \alpha_h}\right)^2 \geq R_\ell - \frac{1}{2}\left(\frac{y_\ell}{\omega_\ell^\eta \alpha_h}\right)^2, \tag{21}$$

$$R_\ell - \frac{1}{2}\left(\frac{y_\ell}{\omega_\ell^\eta \alpha_\ell}\right)^2 \geq 0, \tag{22}$$

$$R_h - \frac{1}{2}\left(\frac{y_h}{\omega_h^\eta \alpha_h}\right)^2 \geq 0, \tag{23}$$

where we have used the relation $e = \frac{y}{\omega^\eta \alpha}$; (20) and (21) are the incentive compatibility constraints for the low and high types, respectively. (22) and (23) are the individual rationality constraints for the low and high types, respectively.

It is straightforward to check that the first-best solution does not satisfy the high-type's incentive compatibility constraint, (21). Under first-best solution, the LHS of (21) equals zero, while the RHS is $\frac{1}{2}\left(\frac{y_{fb,\ell}}{w_{fb,\ell}^\eta}\right)^2\left(\frac{1}{\alpha_\ell^2} - \frac{1}{\alpha_h^2}\right) > 0$; hence, the first-best is not implementable.

Some straightforward but lengthy arguments show that, only the incentive compatibility constraint for the high-type, (21), and the participation constraint for the low-type, (22), bind. These binding constraints imply:

$$R_\ell = \frac{1}{2}\left(\frac{y_\ell}{\omega_\ell^\eta \alpha_\ell}\right)^2, \quad \text{and} \quad R_h = \frac{1}{2}\left(\frac{y_h}{\omega_h^\eta \alpha_h}\right)^2 + \frac{1}{2}\left(\frac{y_\ell}{\omega_\ell^\eta \alpha_\ell}\right)^2 - \frac{1}{2}\left(\frac{y_\ell}{\omega_\ell^\eta \alpha_h}\right)^2.$$

Substituting R_ℓ and R_h in (19), write the principal's problem as:

$$\max_{\{\omega_\tau, y_\tau\}_{\tau\in\{\ell,h\}}} p\left[y_h - \frac{1}{2}\left(\frac{y_h}{\omega_h^\eta \alpha_h}\right)^2 - \frac{1}{2}\left(\frac{y_\ell}{\omega_\ell^\eta \alpha_\ell}\right)^2 + \frac{1}{2}\left(\frac{y_\ell}{\omega_\ell^\eta \alpha_h}\right)^2 - \omega_h\right] + (1-p)\left[y_\ell - \frac{1}{2}\left(\frac{y_\ell}{\omega_\ell^\eta \alpha_\ell}\right)^2 - \omega_\ell\right]. \tag{24}$$

The first-order conditions with respect to y_h and y_ℓ yield, respectively:

$$y_h = (\omega_h^\eta \alpha_h)^2, \qquad \text{and} \qquad y_\ell = \gamma(\omega_\ell^\eta \alpha_\ell)^2, \tag{25}$$

where $\gamma \equiv \frac{1-p}{1-p\frac{\alpha_\ell^2}{\alpha_h^2}} < 1$. Using (25), the principal's problem, (24), can be written as:

$$\max_{\{\omega_\tau\}_{\tau\in\{\ell,h\}}} \; p\left[\frac{1}{2}\left(\omega_h^\eta \alpha_h\right)^2 + \frac{1}{2}\left(\frac{\gamma\omega_\ell^\eta \alpha_\ell^2}{\alpha_h}\right)^2 - \omega_h\right] + (1-p)\left[\gamma\left(\omega_\ell^\eta \alpha_\ell\right)^2 - \omega_\ell\right] - \frac{1}{2}\gamma^2\left(\omega_\ell^\eta \alpha_\ell\right)^2.$$

The optimality conditions yield:

$$\omega_{ur.sb,h} = (\eta\alpha_h^2)^{\frac{1}{1-2\eta}} = \omega_{fb,h}, \qquad \text{and} \qquad \omega_{ur.sb,\ell} = (\gamma\eta\alpha_\ell^2)^{\frac{1}{1-2\eta}} < \omega_{fb,\ell},$$

where the last relation follows since $\gamma < 1$ and the subscript $ur.sb$ refers to the optimal contract solution. Substituting $\omega_{ur.sb,h}$ and $\omega_{ur.sb,\ell}$ in (25) gives the prescribed output levels:

$$y_{ur.sb,h} = (\alpha_h\eta^\eta)^{\frac{2}{1-2\eta}} = y_{fb,h}, \quad \text{and} \quad y_{ur.sb,\ell} = \gamma^{\frac{1}{1-2\eta}}\left(\alpha_\ell\eta^\eta\right)^{\frac{2}{1-2\eta}} < y_{fb,\ell}.$$

The agents' efforts under the optimal contract are (since $e_{ur.sb,\tau} = \frac{y_{ur.sb,\tau}}{\omega^\eta_{ur.sb,\tau}\alpha_\tau}$):

$$e_{ur.sb,h} = (\alpha_h\eta^\eta)^{\frac{1}{1-2\eta}} = e_{fb,h}, \quad \text{and} \quad e_{ur.sb,\ell} = \gamma^{\frac{1-\eta}{1-2\eta}}\left(\alpha_\ell\eta^\eta\right)^{\frac{1}{1-2\eta}} < e_{fb,\ell}. \qquad \textbf{Q.E.D.}$$

Comparison of optimal share contract and unrestricted second-best contract. From the solution to the optimal share contract, (3) – (5), we have: $\delta_{sc} = \frac{1}{2}$, $\omega_{sc,\tau} = \left(\frac{1}{2}\right)^{\frac{1}{1-2\eta}}\left(\eta\alpha_\tau^2\right)^{\frac{1}{1-2\eta}}$ and $e_{sc,\tau} = \left(\frac{1}{2}\right)^{\frac{1-\eta}{1-2\eta}}\left(\eta^\eta\alpha_\tau\right)^{\frac{1}{1-2\eta}}$. The principal's surplus under the optimal share contract for each type of agent is:

$$V_{sc,\tau} = \frac{1}{4}\left(\frac{1}{2}\eta\alpha_\tau^2\right)^{\left(\frac{2\eta}{1-2\eta}\right)}\alpha_\tau^2 - \left(\frac{1}{2}\eta\alpha_\tau^2\right)^{\left(\frac{1}{1-2\eta}\right)},$$

while, under the second-best contract, they are:

$$\begin{aligned} V_{ur.sb,h} &= \frac{1}{2}\left(\eta^\eta\alpha_h\right)^{\frac{2}{1-2\eta}} - \frac{1}{2}\gamma^{\frac{2-2\eta}{1-2\eta}}\left(1-\frac{\alpha_\ell^2}{\alpha_h^2}\right)\left(\eta^\eta\alpha_\ell\right)^{\frac{2}{1-2\eta}} - \left(\eta\alpha_h^2\right)^{\frac{1}{1-2\eta}}, \\ \text{and} \quad V_{ur.sb,\ell} &= \gamma^{\frac{1}{1-2\eta}}\left(1-\frac{\gamma}{2}\right)\left(\eta^\eta\alpha_\ell\right)^{\frac{2}{1-2\eta}} - \left(\gamma\eta\alpha_\ell^2\right)^{\frac{1}{1-2\eta}}. \end{aligned}$$

Explicit comparison of the principal's expected surplus under the different contracts is not informative, so we compare the surpluses numerically. We assume the following parameters:[16] $\alpha_h = 1$, $\alpha_\ell = 0.75$ and $p = 0.4$. Fig. 5 shows the expected surpluses under different contracts for different values of $\eta \in [0.05, 0.45]$. As expected, the implemented optimal share contract outcome is inferior to the optimal contract. ■

Verification of the second-order conditions for the optimal share contract solutions: Stochastic output case. In order to ensure that the solutions $(\tilde{\omega}_{sc,\tau}, \tilde{\delta}_{sc,\tau})$ are indeed maximizers, rather minimizers or saddle points, we need to check that the Hessian of V is negative semi-definite (Theorem 17.8 of Simon and Blume (1994)). So in the following

[16]Other parameter specifications give similar results.

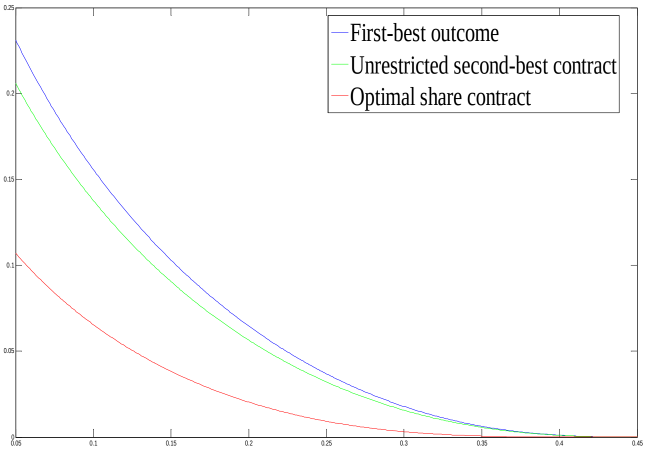

Fig 5: Principal's expected surplus under different contracts

derivations it is enough to show that the final expression will be non-negative, along with our verification of negative signs of the first two expressions:

$$\frac{\partial^2 V}{\partial \omega^2} = 2\eta(2\eta-1)\delta(1-\delta)\alpha_\tau^2\omega^{2\eta-2} < 0, \quad \text{(since } \eta < 1/2\text{)}$$
$$\frac{\partial^2 V}{\partial \delta^2} = -2\alpha_\tau^2\omega^{2\eta} < 0,$$
$$\frac{\partial^2 V}{\partial \omega^2}\frac{\partial^2 V}{\partial \delta^2} - \Big(\frac{\partial^2 V}{\partial \omega \partial \delta}\Big)^2 = 4\eta\alpha_\tau^4\omega^{4\eta-2}\underbrace{\big[-(2\eta-1)\delta(1-\delta)-\eta(1-2\delta)^2\big]}_{\text{sign ?}}.$$

The last expression is non-negative if and only if

$$\delta(1-\delta) \geq \frac{1}{2+(1/\eta)}. \tag{26}$$

It is easy to see from (10) and (11) that $\tilde{\delta}_{sc,\tau} \in (0, 1/2)$. Also, $\delta(1-\delta) - \frac{1}{2+(1/\eta)} > 0$ at $\delta = 1/2$ for $\eta < 1/2$, so condition (26) is satisfied. Further, $\delta(1-\delta)$ is strictly increasing in $\delta \in [0, 1/2)$, and given that at $\delta = 0$ condition (26) is violated, there must be some critical $\bar{\delta} \in (0, 1/2)$ such that condition (26) will hold for all $\delta \in [\bar{\delta}, 1/2]$ and will be violated for $\delta < \bar{\delta}$. One can conclude therefore the optimal share contract, $\delta_{sc,\tau}$, must lie somewhere in the interval $[\bar{\delta}, 1/2)$. (It should be noted that the difficulty of verifying the non-negativity condition (26) using the first-order conditions is that the solutions are not explicit. But since the principal's maximization problem will have solutions that are interior, the second-order conditions, in particular condition (26), must be satisfied.) ■

Proof of Proposition 3. (i) First we claim that if $\frac{\partial \tilde{\omega}_{sc,\tau}}{\partial \alpha_\tau} \leq 0$ then it must be that $\frac{\partial \tilde{\delta}_{sc,\tau}}{\partial \alpha_\tau} < 0$. To see this, go back to (10). As α_τ increases and so $\tilde{\omega}_{sc,\tau}$ decreases (or stays unchanged), $\alpha_\tau^2 \tilde{\omega}_{sc,\tau}^{2\eta-1}$ would increase (because $\eta < 1/2$). Hence to satisfy (10), it must be that $\tilde{\delta}_{sc,\tau}(1-\tilde{\delta}_{sc,\tau})$ must decrease, i.e., $(1-2\tilde{\delta}_{sc,\tau})\frac{\partial \tilde{\delta}_{sc,\tau}}{\partial \alpha_\tau} < 0$, i.e., $\frac{\partial \tilde{\delta}_{sc,\tau}}{\partial \alpha_\tau} < 0$ (from (11), $(1-2\tilde{\delta}_{sc,\tau}) > 0$).

Next we claim that $\tilde{\omega}_{sc,h} > \tilde{\omega}_{sc,\ell}$. Suppose not, so that $\tilde{\omega}_{sc,h} \leq \tilde{\omega}_{sc,\ell}$ (recall, $\alpha_h > \alpha_\ell$). Then by the above argument, $\tilde{\delta}_{sc,h} < \tilde{\delta}_{sc,\ell}$. By the definition of the optimal share contract, the principal's payoff by offering h type's (ℓ type's) incentives when the employee is actually of h type (ℓ type) must be weakly greater than the payoff if he offers ℓ type's (h type's) incentives:

$$\tilde{\delta}_{sc,h}(1-\tilde{\delta}_{sc,h})\alpha_h^2\tilde{\omega}_{sc,h}^{2\eta}+(1-\tilde{\delta}_{sc,h})\bar{\epsilon}-\tilde{\omega}_{sc,h}\geq\tilde{\delta}_{sc,\ell}(1-\tilde{\delta}_{sc,\ell})\alpha_h^2\tilde{\omega}_{sc,\ell}^{2\eta}+(1-\tilde{\delta}_{sc,\ell})\bar{\epsilon}-\tilde{\omega}_{sc,\ell}, \quad (27)$$

$$\tilde{\delta}_{sc,\ell}(1-\tilde{\delta}_{sc,\ell})\alpha_\ell^2\tilde{\omega}_{sc,\ell}^{2\eta}+(1-\tilde{\delta}_{sc,\ell})\bar{\epsilon}-\tilde{\omega}_{sc,\ell}\geq\tilde{\delta}_{sc,h}(1-\tilde{\delta}_{sc,h})\alpha_\ell^2\tilde{\omega}_{sc,h}^{2\eta}+(1-\tilde{\delta}_{sc,h})\bar{\epsilon}-\tilde{\omega}_{sc,h}. \quad (28)$$

Rewrite (27) and (28), respectively, as follows:

$$\tilde{\delta}_{sc,\ell}(1-\tilde{\delta}_{sc,\ell})\alpha_h^2\tilde{\omega}_{sc,\ell}^{2\eta}-\tilde{\delta}_{sc,h}(1-\tilde{\delta}_{sc,h})\alpha_h^2\tilde{\omega}_{sc,h}^{2\eta}\leq[(1-\tilde{\delta}_{sc,h})\bar{\epsilon}-\tilde{\omega}_{sc,h}]-[(1-\tilde{\delta}_{sc,\ell})\bar{\epsilon}-\tilde{\omega}_{sc,\ell}], \quad (29)$$

$$\tilde{\delta}_{sc,\ell}(1-\tilde{\delta}_{sc,\ell})\alpha_\ell^2\tilde{\omega}_{sc,\ell}^{2\eta}-\tilde{\delta}_{sc,h}(1-\tilde{\delta}_{sc,h})\alpha_\ell^2\tilde{\omega}_{sc,h}^{2\eta}\geq[(1-\tilde{\delta}_{sc,h})\bar{\epsilon}-\tilde{\omega}_{sc,h}]-[(1-\tilde{\delta}_{sc,\ell})\bar{\epsilon}-\tilde{\omega}_{sc,\ell}]. \quad (30)$$

Clearly, (29) and (30) together yield a contradiction, given that $\alpha_h > \alpha_\ell$.

The argument just completed shows that $\tilde{\omega}_{sc,\tau}$ is (strictly) increasing in α_τ. So, as α_τ increases, $\alpha_\tau^2\tilde{\omega}_{sc,\tau}^{2\eta}$ would increase, and to satisfy (11) it must be that $\frac{\partial\tilde{\delta}_{sc,\tau}}{\partial\alpha_\tau} > 0$.

(ii) By setting $\bar{\epsilon} = 0$ in (11), it follows that $\tilde{\delta}_{sc,\tau} = 1/2$ that, along with (10), imply that $\tilde{\omega}_{sc,\tau} = \omega_{sc,\tau}$ as in (4). **Q.E.D.**

Proof of Lemma 2. Agent's self-selection according to types requires:

$$U_\ell(\hat{y}<\bar{y})\geq U_\ell(\hat{y}\geq\bar{y}), \qquad U_h(\hat{y}\geq\bar{y})\geq U_h(\hat{y}<\bar{y}),$$

where $U_\tau(.)$ is the expected utility of the type τ agent. It is easy to check that the (ex-ante) individual rationality constraints will be fulfilled if the agent projects a target according to her true type, for the incentive mechanism (14). Also, limited liability is satisfied.

Low target announcement and equilibrium effort choice. If an agent of type τ announces a target $\hat{y} < \bar{y}$, then she is provided with the resources, $\omega = \tilde{\omega}_{sc,\ell}$, and she solves the following problem:

$$\max_e \ \mathbb{E}_\epsilon\left[\tilde{\delta}_{sc,\ell}\left(\tilde{\omega}_{sc,\ell}^\eta\alpha_\tau e+\epsilon\right)-\frac{1}{2}e^2\right],$$

inducing her to exert the effort level,

$$e_\tau^*(\hat{y}<\bar{y})=\tilde{\delta}_{sc,\ell}\ \tilde{\omega}_{sc,\ell}^\eta\ \alpha_\tau, \qquad \tau=\ell,h.$$

The low-type agent, thus, exerts her optimal (share contract) effort: $e_\ell^*(\hat{y}<\bar{y})=\tilde{e}_{sc,\ell}$.

Accordingly, the expected utility of agent type τ is

$$U_\tau(\hat{y}<\bar{y})=\frac{1}{2}\tilde{\delta}_{sc,\ell}^2\ \tilde{\omega}_{sc,\ell}^{2\eta}\ \alpha_\tau^2+\tilde{\delta}_{sc,\ell}\ \bar{\epsilon}.$$

High target announcement and equilibrium effort choice. Now consider an agent of type τ who announces a target $\hat{y} \geq \bar{y}$ and is given the resources, $\omega = \tilde{\omega}_{sc,h}$; so her ex-post payoff, given any effort level e, is

$$\begin{cases} \tilde{\delta}_{sc,h}y-\min\{(1-\tilde{\delta}_{sc,h})[\bar{y}-y],\tilde{\delta}_{sc,h}y\}-\frac{1}{2}e^2, & \text{if} \quad y<\bar{y} \\ \tilde{\delta}_{sc,h}y-\frac{1}{2}e^2, & \text{if} \quad y\geq\bar{y}, \end{cases}$$

that is,

$$\begin{cases} -\frac{1}{2}e^2, & \text{if} \quad y \leq (1-\tilde{\delta}_{sc,h})\bar{y} \\ y-(1-\tilde{\delta}_{sc,h})\bar{y}-\frac{1}{2}e^2, & \text{if} \quad (1-\tilde{\delta}_{sc,h})\bar{y} \leq y < \bar{y} \\ \tilde{\delta}_{sc,h}y-\frac{1}{2}e^2, & \text{if} \quad y \geq \bar{y}. \end{cases}$$

Different ranges of y correspond to appropriate ranges of ϵ.[17] For instance, $(1-\tilde{\delta}_{sc,h})\bar{y} < y < \bar{y}$ if $(1-\tilde{\delta}_{sc,h})\bar{y} < \tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e+\epsilon < \bar{y} \Leftrightarrow (1-\tilde{\delta}_{sc,h})\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e < \epsilon < \bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e$, the probability of which is $F(\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e)-F((1-\tilde{\delta}_{sc,h})\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e)$.

The agent thus solves the following problem:

$$\begin{aligned} \max_{e}\ U_{\mathcal{T}}(e|\hat{y}\geq\bar{y}) \quad &:= \quad \int_{(1-\tilde{\delta}_{sc,h})\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e}^{\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e} \left\{[\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e+\epsilon]-(1-\tilde{\delta}_{sc,h})\bar{y}\right\} f(\epsilon)d\epsilon \\ &+ \quad \int_{\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e}^{\epsilon^+} \left\{\tilde{\delta}_{sc,h}[\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e+\epsilon]\right\} f(\epsilon)d\epsilon \ -\frac{1}{2}e^2. \end{aligned} \tag{31}$$

Differentiate $U_{\mathcal{T}}(e|\hat{y}\geq\bar{y})$ w.r.t. e, denote it by $\psi(e)$, and set it equal to zero:

$$\begin{aligned} \psi(e) \quad &= \quad \left[\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e+\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e-(1-\tilde{\delta}_{sc,h})\bar{y}\right] f(\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e)[-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}] \\ &\quad -\left[\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e+(1-\tilde{\delta}_{sc,h})\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e-(1-\tilde{\delta}_{sc,h})\bar{y}\right] f\left((1-\tilde{\delta}_{sc,h})\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e\right)[-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}] \\ &\quad +\int_{(1-\tilde{\delta}_{sc,h})\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e}^{\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e} \left\{\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}\right\} f(\epsilon)d\epsilon+\left[\tilde{\delta}_{sc,h}(\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e+\epsilon^+)\right].f(\epsilon^+).0 \\ &\quad -\left[\tilde{\delta}_{sc,h}(\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e+\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e)\right] f(\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e)[-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}] \\ &\quad +\int_{\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e}^{\epsilon^+} \left\{\tilde{\delta}_{sc,h}\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}\right\} f(\epsilon)d(\epsilon)-e \\ &= \quad \tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}\left[F(\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e)-F\big((1-\tilde{\delta}_{sc,h})\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e\big)\right] \\ &\quad +\tilde{\delta}_{sc,h}\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}\left[1-F(\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e)\right]-e \\ &= \quad \tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}\left[\tilde{\delta}_{sc,h}+(1-\tilde{\delta}_{sc,h})F(\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e)-F\left((1-\tilde{\delta}_{sc,h})\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e\right)\right]-e=0, \end{aligned} \tag{32}$$

to solve for the agent's equilibrium effort level, (15), repeated here for convenience:

$$e^*_{\mathcal{T},h}=\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}\left[\tilde{\delta}_{sc,h}+(1-\tilde{\delta}_{sc,h})F(\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e^*_{\mathcal{T},h})-F\left((1-\tilde{\delta}_{sc,h})\bar{y}-\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}e^*_{\mathcal{T},h}\right)\right].$$

The function $\psi(e)$, given by (32), is continuous in e. For any $\bar{y}\geq\tilde{y}_{sc,h}$, let $\hat{e}_{\mathcal{T}}$ be such that

$$\tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}\hat{e}_{\mathcal{T}}=\bar{y}, \tag{33}$$

where $\tilde{y}_{sc,h}$ is as defined in (13).

Verify that

$$\psi(0) \quad = \quad \tilde{\omega}^{\eta}_{sc,h}\alpha_{\mathcal{T}}\left[\tilde{\delta}_{sc,h}(1-F(\bar{y}))+F(\bar{y})-F\big((1-\tilde{\delta}_{sc,h})\bar{y}\big)\right]>0,$$

[17] Note that for $y\leq(1-\tilde{\delta}_{sc,h})\bar{y}$, the limited liability constraint is satisfied as the principal gives zero (not negative) remuneration to the agent.

$$\begin{aligned} \text{and} \quad \psi(\hat{e}_\tau) &= \tilde{\omega}^{\eta}_{sc,h}\alpha_\tau\Big[\tilde{\delta}_{sc,h} + (1-\tilde{\delta}_{sc,h})F(0) - F\big(\underbrace{(1-\tilde{\delta}_{sc,h})\bar{y} - \tilde{\omega}^{\eta}_{sc,h}\alpha_\tau\hat{e}_\tau}_{<0}\big)\Big] - \hat{e}_\tau \\ &= \tilde{\omega}^{\eta}_{sc,h}\alpha_\tau\tilde{\delta}_{sc,h} - \hat{e}_\tau \\ &\leq \frac{\tilde{e}_{sc,h}}{\alpha_h}\alpha_\tau - \tilde{e}_{sc,h}\frac{\alpha_h}{\alpha_\tau} \qquad \left((33) \text{ and } \bar{y} \geq \tilde{y}_{sc,h} \text{ imply } \hat{e}_\tau \geq \frac{\alpha_h\tilde{e}_{sc,h}}{\alpha_\tau}; \text{also use } (12)\right) \\ &\leq 0. \end{aligned}$$

So, by the intermediate-value theorem there exists $0 < e^*_{\tau,h} \leq \hat{e}_\tau, \tau = \ell, h$, such that $\psi(e^*_{\tau,h}) = 0$; these are the optimal efforts satisfying (15).

Further, for $\bar{y} > \tilde{y}_{sc,h}$, it must be that $\psi(\hat{e}_\tau) < 0$ so that $e^*_{\tau,h} < \hat{e}_\tau$, i.e., the deterministic part of output $\tilde{\omega}^{\eta}_{sc,h}\alpha_\tau e^*_{\tau,h} < \bar{y}$.

If $\bar{y} = \tilde{y}_{sc,h}$ then $\hat{e}_h = \tilde{e}_{sc,h}$ and $\psi(\tilde{e}_{sc,h}) = 0$ so that $e^*_{h,h} = \tilde{e}_{sc,h}$ and the high-type agent's deterministic output exactly equals the target. However, the low-type agent may fail to meet the proclaimed high target, as $\psi(\hat{e}_\ell) < 0$ and therefore $e^*_{\ell,h} < \hat{e}_\ell$. **Q.E.D.**

Proof of Proposition 4. If the high-type agent announces a high target, $\hat{y} = \tilde{y}_{sc,h}$, and puts in the optimal share contract effort, $\tilde{e}_{sc,h} = \tilde{\delta}_{sc,h}\tilde{\omega}^{\eta}_{sc,h}\alpha_h$, she meets the target even with the lowest realization of the random shock, i.e., with $\epsilon = 0$.[18] Her payoff then equals $U_h(\hat{y} = \tilde{y}_{sc,h}) = \frac{1}{2}\tilde{\delta}^2_{sc,h}\tilde{\omega}^{2\eta}_{sc,h}\alpha^2_h + \tilde{\delta}_{sc,h}\bar{\epsilon}$. However, her maximal payoff from announcing a low target is $U_h(\hat{y} < \tilde{y}_{sc,h}) = \frac{1}{2}\tilde{\delta}^2_{sc,\ell}\tilde{\omega}^{2\eta}_{sc,\ell}\alpha^2_h + \tilde{\delta}_{sc,\ell}\bar{\epsilon}$, which is strictly less than $U_h(\hat{y} = \tilde{y}_{sc,h})$. Thus, the high-type agent would self-select by announcing $\hat{y} = \tilde{y}_{sc,h}$.

Also, by Lemma 2, the high-type agent will put in the optimal share contract effort level, $\tilde{e}_{sc,h}$, and always achieve the target. **Q.E.D.**

Proof of Proposition 6. Consider (16). For $\bar{y} = \tilde{y}_{sc,h}$, it is straightforward to verify that the high-type agent would self-select by announcing a high target, $\hat{y} = \tilde{y}_{sc,h}$, put in an effort corresponding to the optimal share contract, $\tilde{e}_{sc,h}$, and always achieve the target; this follows from the fact that the contract (16) has higher downward reward adjustment for underperformance than the contract (14), and under the latter contract the high-type agent chose to self-select and put in the optimal share contract effort (by Proposition 4).

Suppose now the low-type agent projects $\hat{y} = \tilde{y}_{sc,h}$ and is given the high-type's resource, $\tilde{\omega}_{sc,h}$. Define e^c as the critical effort that she needs to exert to produce, without fail, her announced target: $\tilde{\omega}^{\eta}_{sc,h}\alpha_\ell e^c = \tilde{y}_{sc,h}$. It can be easily verified that

$$e^c = \tilde{\delta}_{sc,h}\tilde{\omega}^{\eta}_{sc,h}\frac{\alpha^2_h}{\alpha_\ell}. \tag{34}$$

If the low-type agent puts in an effort of at least e^c, her expected utility is

$$U_\ell(e \geq e^c) = \tilde{\delta}_{sc,h}\tilde{\omega}^{\eta}_{sc,h}\alpha_\ell e + \tilde{\delta}_{sc,h}\bar{\epsilon} - \frac{1}{2}e^2.$$

[18] Note that if the principal sets the cut-off target $\bar{y} = \tilde{y}_{sc,h}$, any agent who wants to announce a high target will announce $\hat{y} = \bar{y} = \tilde{y}_{sc,h}$ as there is no additional benefit from announcing a target higher than the cut-off target.

Furthermore,

$$\left.\frac{\partial U_\ell(e>e^c)}{\partial e}\right|_{e\downarrow e^c} = \tilde{\delta}_{sc,h}\tilde{\omega}^{\eta}_{sc,h}\left[\frac{(\alpha_\ell^2-\alpha_h^2)}{\alpha_\ell}\right] < 0,$$

so the low-type agent should never choose an effort level in excess of e^c.

Suppose the low-type agent exerts an effort $e < e^c$. Then her ex-post payoff is:

$$\begin{cases} \tilde{\delta}_{sc,h}y - \frac{1}{2}e^2, & \text{if} \quad y \geq \bar{y} \\ \gamma y - \frac{1}{2}e^2, & \text{if} \quad y < \bar{y}. \end{cases}$$

Note that $y < \bar{y} \Leftrightarrow \tilde{\omega}^{\eta}_{sc,h}\alpha_\ell e + \epsilon < \bar{y} \Leftrightarrow \epsilon < \bar{y} - \tilde{\omega}^{\eta}_{sc,h}\alpha_\ell e$, so the probability that the low-type agent falls short of the projected high target, $\hat{y} = \tilde{y}_{sc,h}$,[19] is $F(\tilde{y}_{sc,h} - \tilde{\omega}^{\eta}_{sc,h}\alpha_\ell e)$. The low-type agent, thus, solves:

$$\max_{e<e^c} U_\ell(e|\hat{y} = \tilde{y}_{sc,h}) := \int_0^{\tilde{y}_{sc,h}-\tilde{\omega}^{\eta}_{sc,h}\alpha_\ell e} \gamma y f(\epsilon)d\epsilon + \int_{\tilde{y}_{sc,h}-\tilde{\omega}^{\eta}_{sc,h}\alpha_\ell e}^{\epsilon^+} \tilde{\delta}_{sc,h} y f(\epsilon)d\epsilon \; - \frac{1}{2}e^2. \tag{35}$$

Differentiating (35) w.r.t. e yields:

$$\frac{\partial U_\ell}{\partial e} = \tilde{\delta}_{sc,h}\tilde{\omega}^{\eta}_{sc,h}\alpha_\ell + (\tilde{\delta}_{sc,h}-\gamma)\tilde{\omega}^{\eta}_{sc,h}\alpha_\ell\left[\tilde{y}_{sc,h}f(\tilde{y}_{sc,h}-\tilde{\omega}^{\eta}_{sc,h}\alpha_\ell e) - F(\tilde{y}_{sc,h}-\tilde{\omega}^{\eta}_{sc,h}\alpha_\ell e)\right] - e. \tag{36}$$

Now by letting $\gamma \to 0$, (36) simplifies as follows:

$$\left.\frac{\partial U_\ell}{\partial e}\right|_{e<e^c} = \tilde{\delta}_{sc,h}\tilde{\omega}^{\eta}_{sc,h}\alpha_\ell\big[1 - F(\tilde{y}_{sc,h}-\tilde{\omega}^{\eta}_{sc,h}\alpha_\ell e) + \tilde{y}_{sc,h}f(\tilde{y}_{sc,h}-\tilde{\omega}^{\eta}_{sc,h}\alpha_\ell e)\big] - e. \tag{37}$$

And further,

$$\left.\frac{\partial U_\ell}{\partial e}\right|_{e\uparrow e^c} = \tilde{\delta}_{sc,h}\tilde{\omega}^{\eta}_{sc,h}\frac{\alpha_\ell^2(1+\tilde{y}_{sc,h}f(0)) - \alpha_h^2}{\alpha_\ell}. \tag{38}$$

Let us now invoke the assumption that the error term, ϵ, follows a uniform distribution so that $f(\epsilon) = \frac{1}{\epsilon^+}$, $F(\epsilon) = \frac{\epsilon}{\epsilon^+}$ and $\mathbb{E}(\epsilon) = \frac{\epsilon^+}{2}$. Let us check the suitability of effort choice $e < e^c$. Using (38), it can be verified that

$$\left.\frac{\partial U_\ell}{\partial e}\right|_{e\uparrow e^c} = \frac{\tilde{\delta}_{sc,h}\tilde{\omega}^{\eta}_{sc,h}}{\alpha_\ell}\left[\alpha_\ell^2\frac{\tilde{y}_{sc,h}+\epsilon^+}{\epsilon^+} - \alpha_h^2\right] \underbrace{>}_{\text{(by (17))}} 0.$$

So in the left-neighborhood of the critical effort e^c, the low-type agent's expected utility is increasing in effort. We had earlier ruled out the possibility of low-type choosing an effort exceeding e^c. Thus, the critical effort e^c is locally an equilibrium choice for the low-type agent after projecting $\hat{y} = \tilde{y}_{sc,h}$.

Next, we check whether any other effort level in $[0, e^c)$ could possibly constitute the low-type agent's equilibrium response. From (37), we have

$$\left.\frac{\partial U_\ell}{\partial e}\right|_{e=0} = \tilde{\delta}_{sc,h}\tilde{\omega}^{\eta}_{sc,h}\alpha_\ell\big[\underbrace{1-F(\tilde{y}_{sc,h})}_{>0} + \underbrace{\tilde{y}_{sc,h}f(\tilde{y}_{sc,h})}_{>0}\big] > 0.$$

[19] Recall that any agent who wants to project a high target will announce $\hat{y} = \bar{y} = \tilde{y}_{sc,h}$.

Hence, it is not optimal for the low-type agent to put in effort $e = 0$. This is true irrespective of any assumption on the error distribution (such as the 'uniform' distribution assumption).

The second-order condition for an interior solution is $\left.\frac{\partial^2 U_\ell}{\partial e^2}\right|_{0<e<e^c} = \frac{\tilde{\delta}_{sc,h}\tilde{\omega}_{sc,h}^{2\eta}\alpha_\ell^2}{\epsilon^+} - 1 < 0$, i.e., $\tilde{\delta}_{sc,h}\tilde{\omega}_{sc,h}^{2\eta}\alpha_\ell^2 < \epsilon^+$, which is ruled out by condition (17) (because $\epsilon^+ < \tilde{y}_{sc,\ell}$). Hence, no interior effort level in $(0, e^c)$ can be an equilibrium for the low-type agent.

Hence, the low-type agent either self-selects and announces a low target, $\hat{y} < \tilde{y}_{sc,h}$, or if she announces a high target, $\hat{y} = \tilde{y}_{sc,h}$, she will surely achieve the projected high target. In either case, from the principal's perspective, the outcome is at least as good as the optimal share contract outcome. **Q.E.D.**

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