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Procedia Engineering 199 (2017) 2366–2371

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

On surface wave fields arising in soil-structure interaction problems

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Abstract

The paper aims at generalization of the specialized formulation, originally developed for the surface wave fields induced by prescribed surface stresses. We extend this formulation to soil-structure interaction problems with unknown contact stresses and internal sources. The problem for an internal source embedded in an elastic half-plane is reduced to that for prescribed surface stresses by considering the point source solution for an unbounded medium. In this case the sub-problems corresponding to normal and tangential stresses assume a separate treatment. Then, we analyze interaction of an elastic half-plane with a one degree of freedom mass-spring system. The focus is on a near-resonant regime investigated by a perturbation technique.

Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: Rayleigh wave, internal source, soil-structure interaction. .

1. Introduction

A specialized formulation for the Rayleigh wave induced by surface stresses was first proposed in [1] using the symbolic operator Lur'e method, see e.g. [2]. It was then rederived in [3] by a slow-time perturbation of the homogeneous Rayleigh wave field obtained in [4], see also [5–8]. The specialized formulation includes elliptic equations for the Lamé elastic potentials over the interior of the half-plane, together with a hyperbolic equation on the surface, governing Rayleigh wave propagation. The established methodology was then extended to address the effects of 3D wave motion, thin coating layers, mixed boundary conditions, anisotropy and interfaces, see [9–13]. The developed approach allows separation of the contribution of the Rayleigh poles to the overall dynamic response, and is therefore effective when the Rayleigh wave field is dominant compared to that of the bulk waves, e.g. in the far-field zone. Another important class problems allowing efficient use of the specialized Rayleigh wave model is concerned with the near-resonant regimes of moving loads on an elastic half-space, see [14–16].

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1877-7058
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.253

At the same time several important real world problems in soil-structure interaction cannot be tackled started from the aforementioned setup oriented to prescribed surface stresses only. In particular, modelling of the Rayleigh wave fields arising in presence of both surface and underground structures motivates generalisation of the existing methodology.

As model examples, we consider plane harmonic problems for a semi-infinite elastic medium with an embedded internal source and also with a one degree of freedom mass-spring system system attached to the surface. The problem for a source is reduced to earlier studied problems for prescribed surface stresses by using the associated point force solution for an infinite medium. In this case we arrive at two hyperbolic equation on the surface for both Lamé potentials.

General treatment of dynamic interaction of surface structures with environment may involve analysis of pretty sophisticated mixed problems in solid mechanics. In addition, the simplifying point contact assumption fails due to a displacement singularity resulting from a point contact force, e.g. see [17]. Below we sketch a perturbation scheme for a near-resonant regime of the considered mass-spring system. As might be expected, at leading order the external force can be transmitted to the half-plane as it is assumed in the original model for the Rayleigh wave. However, at higher orders, the contribution of bulk waves cannot be ignored.

2. A specialized formulation for the Rayleigh wave field

First, discuss briefly an explicit model for the Rayleigh wave on a linearly elastic, isotropic half-plane, given by $-\infty < x_1 < \infty$, $0 \le x_2 < \infty$, see e.g. [3]. For this problem, the original equations of motion may be written in terms of the plane Lamé elastic potentials ϕ and ψ as

$$c_1^2 \Delta\phi - \ddot{\phi} = 0, \qquad c_2^2 \Delta\psi - \ddot{\psi} = 0, \tag{1}$$

where Δ is the Laplace operator in x_1 and x_2, dot indicates a time derivative, and c_1 and c_2 are the longitudinal and transverse wave speeds, respectively. The boundary conditions on the surface $x_2 = 0$ are imposed in the form of prescribed stresses

$$\begin{aligned} \sigma_{12} &= \mu\left(2\frac{\partial^2\phi}{\partial x_1 \partial x_2} + \frac{\partial^2\psi}{\partial x_1^2} - \frac{\partial^2\psi}{\partial x_2^2}\right) = P_1(x_1, t), \\ \sigma_{22} &= \lambda\frac{\partial^2\phi}{\partial x_1^2} + (\lambda + 2\mu)\frac{\partial^2\phi}{\partial x_2^2} + 2\frac{\partial^2\psi}{\partial x_1 \partial x_2} = P_2(x_2, t), \end{aligned} \tag{2}$$

where λ and μ are the Lamé parameters. The eigensolution of (1) and (2) at $P_1 = P_2 = 0$ can be presented through a single plane harmonic function [4]. In this case the displacements

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_2}, \qquad u_2 = \frac{\partial\phi}{\partial x_2} + \frac{\partial\psi}{\partial x_1} \tag{3}$$

become

$$\begin{aligned} u_1(x_1, x_2, t) &= \frac{\partial\phi(x_1, \alpha_R x_2, t)}{\partial x_1} - \frac{1+\beta_R^2}{2}\,\frac{\partial\phi(x_1, \beta_R x_2, t)}{\partial x_1}, \\ u_2(x_1, x_2, t) &= \frac{\partial\phi(x_1, \alpha_R x_2, t)}{\partial x_2} - \frac{2}{1+\beta_R^2}\,\frac{\partial\phi(x_1, \beta_R x_2, t)}{\partial x_2}, \end{aligned} \tag{4}$$

where ϕ is an arbitrary plane harmonic function in its first two arguments satisfying the wave equation

$$\frac{\partial^2\phi}{\partial x_1^2} - \frac{1}{c_R^2}\ddot{\phi} = 0, \tag{5}$$

for more details see [18], and

$$\alpha_R = \sqrt{1 - \frac{c_R^2}{c_1^2}}, \qquad \beta_R = \sqrt{1 - \frac{c_R^2}{c_2^2}}, \tag{6}$$

with c_R denoting the Rayleigh wave speed, being a unique solution of

$$4\alpha_R\beta_R - (1+\beta_R^2)^2 = 0. \tag{7}$$

In this case the elliptic equations for the potentials ϕ and ψ can be written as

$$\frac{\partial^2\phi}{\partial x_2^2} + \alpha_R^2\frac{\partial^2\phi}{\partial x_1^2} = 0, \qquad \frac{\partial^2\psi}{\partial x_2^2} + \beta_R^2\frac{\partial^2\psi}{\partial x_1^2} = 0. \tag{8}$$

In addition, the potentials ϕ and ψ are in fact harmonic conjugates, see [4]

$$\psi = \frac{2\alpha_R}{1+\beta_R^2}\,\phi^*, \tag{9}$$

with the asterisk denoting a harmonic conjugate function.

The cases of vertical and horizontal loading are treated separately within the specialized framework for the Rayleigh wave in question, see [3,12]. The first one ($P_1 = 0, P_2 \neq 0$) results in a hyperbolic equation for the longitudinal potential along the surface $x_2 = 0$ given by

$$\frac{\partial^2\phi}{\partial x_1^2} - \frac{1}{c_R^2}\ddot{\phi} = \frac{(1+\beta_R^2)P_2}{2\mu B}, \tag{10}$$

whereas in the second case ($P_1 \neq 0, P_2 = 0$) we have a similar equation for the transverse potential. It is

$$\frac{\partial^2\psi}{\partial x_1^2} - \frac{1}{c_R^2}\ddot{\psi} = -\frac{(1+\beta_R^2)P_1}{2\mu B}. \tag{11}$$

In above B is a material constant defined as

$$B = \frac{\alpha_R}{\beta_R}(1-\beta_R^2) + \frac{\beta_R}{\alpha_R}(1-\alpha_R^2) - 1 + \beta_R^4. \tag{12}$$

Thus, the elastic potentials, being solutions of the elliptic equations (8), should satisfy the condition (9) along with the boundary conditions on the surface $x_2 = 0$, given by the hyperbolic equations (10) and (11).

3. Internal point source

Let us now study the Rayleigh wave field induced by an embedded time-harmonic point source, see Fig. 1, located on the x_2 axis at depth a from the surface.

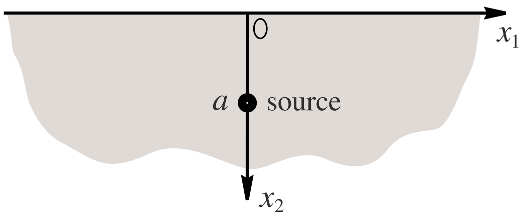

Fig. 1: An internal source.

On employing the superposition principle, first consider radiation from a point source in an unbounded plane. Let us focus, for example, on the longitudinal elastic potential ϕ, which is given by the Green function

$$\phi = \frac{i}{4k^2}H_0^{(1)}(kr) \tag{13}$$

where ω is frequency, $k = \omega/c_1$ is the wave number, $r = \sqrt{x_1^2 + x_2^2}$ is the polar radius, and $H_0^{(1)}$ is a Hankel function of the first kind, see e.g. [17]. Here and below the factor $\exp(-i\omega t)$ is omitted.

The associated stress components are

$$\begin{aligned} s_{12} &= \frac{i\mu}{2}\,\frac{x_1 x_2}{x_1^2 + x_2^2} H_2^{(1)}\left(k\sqrt{x_1^2 + x_2^2}\right), \\ s_{22} &= \frac{i}{4}\left[\frac{2\mu}{k}\,\frac{x_2^2 - x_1^2}{(x_1^2 + x_2^2)^{3/2}}\, H_1^{(1)}\left(k\sqrt{x_1^2 + x_2^2}\right) - \frac{\lambda x_1^2 + (\lambda + 2\mu)x_2^2}{x_1^2 + x_2^2} H_0^{(1)}\left(k\sqrt{x_1^2 + x_2^2}\right)\right]. \end{aligned} \tag{14}$$

Now, we are in position to formulate the non-homogeneous problem for a half-plane without an embedded source, i.e.

$$\sigma_{12} = -s_{12}\big|_{x_2=a}, \qquad \sigma_{22} = -s_{22}\big|_{x_2=a}. \tag{15}$$

On introducing the dimensionless quantities

$$\xi_1 = \frac{x_1}{a}, \qquad \xi_2 = \frac{x_2}{a}, \qquad k_1 = ka = \frac{\omega a}{c_1}, \tag{16}$$

the loading in conditions (2) take the form

$$\begin{aligned} P_1 &= -\frac{i\mu}{2}\,\frac{\xi_1}{\xi_1^2 + 1} H_2^{(1)}\left(k_1\sqrt{\xi_1^2 + 1}\right), \\ P_2 &= \frac{i}{4}\left[\frac{2\mu}{k_1}\,\frac{\xi_1^2 - 1}{(\xi_1^2 + 1)^{3/2}}\, H_1^{(1)}\left(k_1\sqrt{\xi_1^2 + 1}\right) + \left(\lambda + \frac{2\mu}{\xi_1^2 + 1}\right) H_0^{(1)}\left(k_1\sqrt{\xi_1^2 + 1}\right)\right], \end{aligned} \tag{17}$$

Consider in greater detail the effect of a vertical load ($P_1 = 0, P_2 \neq 0$), for which the hyperbolic equation (10) becomes

$$\frac{d^2\phi}{d\xi_1^2} + k_R^2\phi = \frac{(1 + \beta_R^2)a^2 P_2}{2\mu B}, \tag{18}$$

with P_2 given by (17) and $k_R = \dfrac{\omega a}{c_R}$. Its solution may be written as

$$\phi(\xi_1, 0, t) = \frac{ia^2(1 + \beta_R^2)}{2k_R B}\, I(\omega)\sin(k_R\xi_1), \tag{19}$$

where

$$I(\omega) = \int_0^\infty \left[\frac{\eta^2 - 1}{k_1(\eta^2 + 1)^{3/2}}\, H_1^{(1)}\left(k_1\sqrt{\eta^2 + 1}\right) + \left(\frac{\nu}{1 - 2\nu} + \frac{1}{\eta^2 + 1}\right) H_0^{(1)}\left(k_1\sqrt{\eta^2 + 1}\right)\right]\cos(k_R\eta)\, d\eta, \tag{20}$$

where ν is the Poisson's ratio. Then, we solve the first elliptic equation in (8) subject to the boundary condition (19). The result is

$$\phi(\xi_1, \xi_2, t) = \frac{ia^2(1 + \beta_R^2)}{2k_R B}\, I(\omega)\sin(k_R\xi_1)\exp(-\alpha_R k_R \xi_2). \tag{21}$$

On employing the relations (9) and $(8)_2$, the related transverse potential ψ is obtained as

$$\psi(\xi_1, \xi_2, t) = -\frac{ia^2\alpha_R}{k_R B}\, I(\omega)\sin(k_R\xi_1)\exp(-\beta_R k_R \xi_2). \tag{22}$$

The case of a horizontal load ($P_1 \neq 0, P_2 = 0$), in which P_1 is given by (17), is treated similarly.

4. Mass-spring system

Consider a one degree of freedom mass-spring system attached to the surface of an elastic half-plane, see Fig. 2, starting from the simplest equations of motion

$$m\ddot{w} + cw = F - P_2, \tag{23}$$

where $w = w(x_1, t)$ is displacement, m, c, and $F(x_1, t)$ are distributed mass, stiffness, and external load, respectively, and $P_2(x_1, t)$ is the unknown contact normal stress.

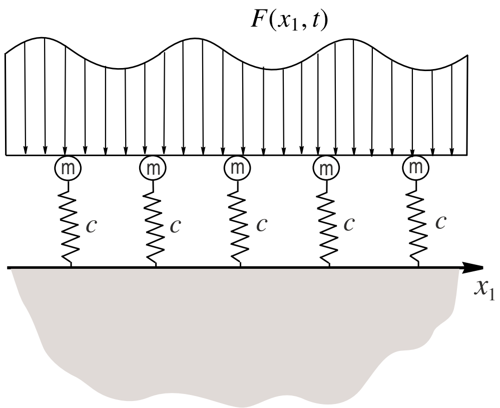

Fig. 2: A mass-spring system.

The continuity conditions are given by $(2)_2$ and also by the formula

$$w = u_2. \tag{24}$$

Study a harmonic near-resonant regime characterized by the small parameter

$$\varepsilon = 1 - \frac{m\omega^2}{c}, \quad |\varepsilon| \ll 1. \tag{25}$$

In this case (23) becomes

$$\varepsilon w = \frac{F - P_2}{c}. \tag{26}$$

It is obvious that $\omega = \sqrt{\frac{c}{m}}$ is the resonant frequency of the mass-spring system. Let us expand the functions $\phi, \psi, u_2, w, \sigma_{22}$ and P_2 in the equations of plane elasticity (1)-(3) and also equations (23) and (24) of this section into asymptotic series as

$$\begin{aligned} &\phi = \phi^{(0)} + \varepsilon\phi^{(1)} + \ldots, && \psi = \psi^{(0)} + \varepsilon\psi^{(1)} + \ldots, && w = w^{(0)} + \varepsilon w^{(1)} + \ldots, \\ &u_2 = u_2^{(0)} + \varepsilon u_2^{(1)} + \ldots, && \sigma_{22} = \sigma_{22}^{(0)} + \varepsilon\sigma_{22}^{(1)} + \ldots, && P_2 = P_2^{(0)} + \varepsilon P_2^{(1)} + \ldots . \end{aligned} \tag{27}$$

At leading order we deduce from (23) that

$$P_2^{(0)} = F,$$

meaning that the external resonant forcing can be transmitted to the half-plane. As a result, the leading order problem is given by the equations in plane elasticity presented in section 2 for the potentials $\phi^{(0)}$ and $\psi^{(0)}$. It is clear that the leading order Rayleigh wave field follows from (10) at $\phi = \phi^{(0)}$ and $P_2 = F$. However, a more accurate evaluation of the surface wave contribution requires the solution of the full plane problem. The reason is that at next order we arrive at

$$P_2^{(1)} = -cw^{(0)} = -cu^{(0)},$$

where $u^{(0)}$ determined from the leading order problems contains not only a Rayleigh wave component, but also that associated with the bulk longitudinal and transverse waves. This is the key peculiarity of soil-structure interaction problems, which is not a feature of the specialized formulation oriented to surface waves induced by prescribed stresses [3].

It is also worth mentioning that the problem considered in this section does not allow a point contact. This is due to a singularity in the vertical displacement caused by a point harmonic load applied to the surface of an elastic half-plane. The effect of distributed contact stresses was analyzed in [19]. A clear link with homogenization techniques should be mentioned, see e.g. [20], addressing a large scale soil-structure interaction problem.

5. Conclusion

The specialised formulation for the Rayleigh wave exposed in [3,12] is extended to soil-structure interaction problems involving dynamic analysis of surface and underground structures. All of them are reduced to the earlier established setup oriented to prescribed surface stresses by using superposition principle and perturbation technique. The developed schemes are not restricted to the presented examples of time-harmonic plane motion. In particular, there is a clear potential for adapting them for a broad range of 3D transient problems inspired by modern industrial applications.

Acknowledgements

The work of NE and BE was supported by the Research Projects of Anadolu University, No: 1408F370, which is gratefully acknowledged.

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